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On the Construction of Some Mercury Standards of Resistance, with a Determination of the Temperature Coefficient of Resistance of Mercury

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II. *On the Construction of Some Mercury Standards of Resistance, with a Determination of the Temperature Coefficient of Resistance of Mercury.*

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[Plates 1–3.]

Introduction.

According to the Order in Council of August 23, 1894, by which certain fundamental electrical units were made legal—

"The ohm which has the value 10^9 in terms of the centimetre, and the second of time, and is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14·4521 grammes in mass, of a constant cross-sectional area, and of a length 106·3 centims.," is one of the units of electrical measurement on which "denomination of standards required for use in trade" is to be based.

The establishment of the National Physical Laboratory rendered it possible to realize this unit in England. With this object, a number of selected tubes of "Verre dur" were obtained from M. Baudin, with the kind assistance of the officials of the Bureau International, while others of Jena 16‴ glass were procured from Messrs. Schott and Co., Jena. The work, however, has necessarily occupied a long time. With the increasing accuracy of electrical measurements it appeared desirable to determine the constants of the tubes so that the probable error should not exceed 1 part in 100,000. Preliminary observations of various kinds were essential to secure success, while the work of organising the various departments of the Laboratory also tended to delay matters; hence it was not until the autumn of 1902 that the real start was made.

The International Ohm.

At a meeting of the British Association Committee at Edinburgh in 1892, the question of establishing identical electric standards in various countries was discussed. Eventually, the following resolutions, amongst others, were agreed to :—

1. That the resistance of a specified column of mercury be adopted as the practical unit of resistance.
2. That 14·4521 grammes of mercury in the form of a column of uniform cross-section, 106·3 centims. in length, at 0° C., be the specified column.

In the construction of such a standard the accurate measurements are, therefore, those of length and mass. For the purposes of this paper the above definition has been extended as implying 106·300 centims.

Theory of Construction.

If L be the length, and s the uniform cross-section of a column of mercury, then its resistance R is given by the equation

$$R = \rho L/s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

where ρ is the specific resistance of the liquid.

If W be the mass of the column, and Δ its density, then

$$R = \rho\Delta\, L^2/W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Since, when $L = 106{\cdot}300$ centims. and $W = 14{\cdot}4521$ grammes, $R = 1$ international ohm, the resistance of any uniform column of mercury is such that $R = 14{\cdot}4521 L^2/(106{\cdot}3)^2\, W$.

The departure of the column from a truly cylindrical form is, however, unavoidable. In a well-chosen standard the tube is conical, preferably uniformly so, but more often in practice it is best likened to a series of truncated cones of various lengths. It is true that unless the section changes by more than 1 per cent., the value of the resistance as calculated from equation (2) is correct to 0·001 per cent., but such a tube is exceptional. In consequence, a correction for this conicality—termed the conical correction—has to be applied.

In the case of tubes varying considerably in cross-section, an elaborate calibration is necessary, and a preliminary set of observations is therefore advisable.

If W be the mass of a small quantity of mercury introduced into the tube for the purpose of calibration, λ the length of the mercury thread when its mid-point is at a distance x from some fixed mark, and Δ the density of mercury, then $s = W/\Delta\lambda$. where s is the mean section of that portion of the tube occupied by the thread.

If the length λ be measured at n points at equal distances along the tube, then the mean section of the whole tube is, to a first approximation $(\Sigma W/\Delta\lambda)/n$.

In general, however, the thread length is not always entirely situated within that portion of the tube afterwards chosen for a resistance standard. Let $a . b . c \ldots$ &c., represent the fractional parts of the length situated within the region mentioned, and L be the length of the resistance standard. Then, if we assume, in the first instance, that the successive portions of the tube occupied by the thread are each uniform in cross-section, the mean section s of the length L will be such that

$$s = (\Sigma \mathrm{W}/\Delta\lambda + a\mathrm{W}/\Delta\lambda_a + b\mathrm{W}/\Delta\lambda_b + \ldots \&c.)/(n + a + b + \ldots),$$

where n represents the number of complete lengths, and λ_a, λ_b, λ_c, &c., those lengths but partly situated within the limits.

The resistance R of a column of mercury at 0° C. of length L and of uniform section s is equal to $\rho\mathrm{L}/s$, where ρ is the specific resistance of mercury.

Similarly the resistance R′ of $(n + a + b + \ldots)$ tubes, each one of constant cross-section, and having a total length L, is given by the equation

$$\mathrm{R}' = \rho\left(\Sigma \frac{\mathrm{L}/n + \ldots}{\mathrm{W}/\Delta\lambda} + \frac{a\mathrm{L}/n + \ldots}{\mathrm{W}/\Delta\lambda_a} + \frac{b\mathrm{L}/n + \ldots}{\mathrm{W}/\Delta\lambda_b} + \ldots\right),$$

whence

$$\frac{\mathrm{R}'}{\mathrm{R}} = \frac{(\Sigma\lambda + a\lambda_a + b\lambda_b + \ldots)(\Sigma 1/\lambda + a/\lambda_a + b/\lambda_b + \ldots)}{(n + a + b + \ldots)^2}.$$

This ratio is the first approximation to the correction for conicality. In the sequel R′/R is denoted by μ'. The calibration was such as to enable five different series to be chosen, the resistance of each series being that of the standard.

There is no justification, however, for assuming that each length L/n is uniform in section, and a correction has still to be applied for their conicality. The nature of this latter is, however, known to a sufficient degree of accuracy from the calibration curve. For, in the case of a tube of slightly conical bore and of uniformly varying section, the mean section for a portion, as determined by the length and mass of a mercury thread, may be regarded as the *actual* section at the mid-point of the thread. Thus, with a conical thread 5 centims. long, and such that the terminal sections differ by 2 per cent., the mid-point of the thread is but 0·02 millim. distant from that section equal to the mean. The irregularities of the 5-centim. lengths are, therefore, approximately portrayed by the calibration curve. The discrepancy is greatest for those positions where the bore is not uniformly conical, and an idea of uniformity is conveyed which is greater than really exists. Such irregularities, however, are insignificant for the present purpose.

The resistance of a conical rod of conducting material being calculable, it becomes easy to evaluate the conical corrections for the 5-centim. lengths, since these latter are of conical structure. The calculation is as follows:—

If dx be a small element of length, and s the corresponding cross-section, then the resistance R of a slightly conical mercury column may be taken as $\rho\int dx/s$. For a uniformly conical column this becomes $\rho\int dx/\pi(r_1 + x \tan\alpha)^2$. If the length of the column be L/n, its resistance R is equal to $\rho L/\pi n r_1 r_2$, where r_1 and r_2 are the radii of the terminal sections (see fig. 1, p. 62). Now the mean section of the cone is $\frac{1}{3}\pi(r_2^2 + r_1^2 + r_1 r_2)$; hence, in the evaluation of μ', the resistance R′ of the column is assumed to be $3\rho L/\pi n(r_2^2 + r_1^2 + r_1 r_2)$; whence

$$R/R' = (r_2^2 + r_1^2 + r_1 r_2)/3r_1 r_2, \quad = 1 + \tfrac{1}{3}(r_2 - r_1)^2/r_1^2$$

approximately, which ratio is hereafter denoted by μ'', and referred to as the conical correction for the 5-centim. lengths.

The values of μ'' for each of the $n + a + b \ldots$ lengths can now be evaluated, since the manner in which the cross-section changes is known. The *mean* value must then be multiplied by the mean value of μ' already calculated. Thus a conical correction designated by μ, and equal to the product $\mu'\mu''$, is obtained for the standard, the bore of which is to be regarded, *not* as a series of uniform tubes, but as one of gradually changing section typified by the calibration curve. The resistance of the column of mercury at 0·0° C. may now be written

$$R = \mu \frac{L^2}{W} \cdot \frac{14{\cdot}4521}{106{\cdot}3^2}.$$

For the evaluation of μ'' a table of the following form was found very useful. In it a_1 and a_2 refer to the terminal sections of the cone, and values of μ'' are tabulated for various ratios of a_1 to a_2. When the difference of sections exceeds 2 per cent., the value of μ'' rises rapidly. In practice the ratio a_1/a_2 did not exceed 1·015 per cent.

TABLE I.

a_1/a_2.	μ''.	a_1/a_2.	μ''.
1·04	1·000131	1·009	1·000007
1·02	1·000033	1·008	1·000005
1·015	1·000019	1·007	1·000004
1·014	1·000016	1·006	1·000003
1·013	1·000014	1·005	1·000002
1·012	1·000012	1·004	1·000001
1·011	1·000010	1·003	1·000001
1·010	1·000008	1·002	1·0000001

Determination of L *and* W.

The ratio of the square of the length of the mercury column at 0·0° C. to its mass may be determined in different ways. The difficulties encountered in any ordinary method of procedure are—(1) the mercury column is not terminated by plane surfaces; (2) the column may not completely fill the tube; (3) films of air and moisture separate the mercury from the glass. With respect to the last of these difficulties, much valuable information is contained in a paper by Dr. G. J. PARKES,* "On the Thickness of Liquid Films formed by Condensation at the Surface of a Solid." It appears that, in all cases where condensation of moisture takes place at temperatures not below the dew-point, the thickness of the surface film of water varies with the substance employed and the conditions of temperature and pressure. The thickness of the water film on glass in saturated vapour at 15° C. is about $13 \cdot 4 \times 10^{-6}$ centim. If, at the commencement, the glass be absolutely dry, an interval of 15 or 16 days elapses before the film attains its maximum thickness, although at the end of 12 hours its thickness is half this maximum. This, it should be remembered, results when the glass is exposed to a saturated atmosphere. To completely remove the film, a very high temperature (probably near 300° C.) is required, but the proportions removed after heating to different temperatures has not apparently been investigated. It appears certain, however, from the data given in the paper, and more especially from the conditions requisite to completely remove the film, that a skin constant in thickness to 1×10^{-6} centim. should be ensured by the operation of the same cycle of temperature. Thus a glass tube chemically cleaned and dried by heating to a temperature approximating to 80° C. (a current of dry air simultaneously passing through the tube) should, on cooling, have a liquid film condensed on its surface, equal in thickness to those of other tubes treated similarly. After the introduction of a mercury column, the film will separate the mercury from the glass, but, since its constancy throughout all operations may be anticipated, it is no longer a source of trouble. This anticipation amounts to almost a certainty if taken in conjunction with the determinations of resistance and of cross-sections dealt with in this paper.

Respecting difficulties (1) and (2), these may be simultaneously disposed of by filling the tube at some definite temperature, and ensuring plane termini by the pressure of pieces of plate glass at the ends; or (1) may be overcome by calculating the length of the column equivalent to the meniscus. When the column is supported horizontally, the meniscus is a portion of a curved surface, which is spherical only if the tube be so narrow that the effect of gravity may be neglected. The radius of nine of the tubes employed averaged 0·43 millim., that of the other two approximated to 0·59 millim. Even with these latter, with which the gravitational effect is more marked, and the extreme point of the meniscus does not therefore lie on the axis of the tube, the equivalent length may be obtained† by assuming the surface as spherical, and the sagitta of the curve as that length of the meniscus observed when looking down at the tube. The calculation is simple.

Let ABD (fig. 2) represent a section of the meniscus by a vertical plane through the axis xy of the tube, the distance OD being the meniscus length or sagitta. Then the volume of the spherical segment is $\frac{1}{2}\pi r^2\text{OD} + \frac{1}{6}\pi\text{OD}^3$, and hence the length of

* 'Proceedings of the Physical Society,' vol. xviii., 1903, p. 410.

† R. T. GLAZEBROOK, 'Phil. Trans.,' A, 1888.

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the mercury column of section πr^2, to which it is equivalent, is $\frac{1}{2}$ OD $(1 + \text{OD}^2/3r^2)$. A measurement of OD and of r enables us, therefore, to calculate the length of a mercury column equivalent to the meniscus.

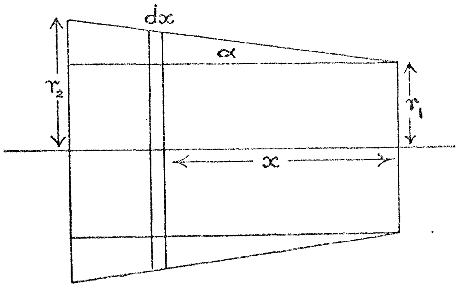

Fig. 1.

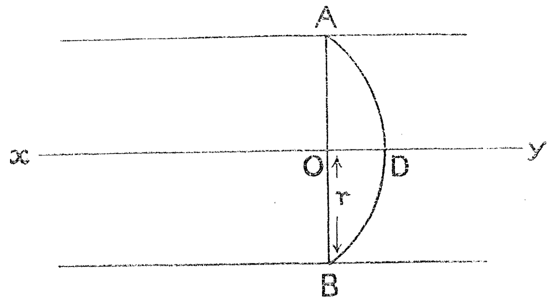

Fig. 2.

It was this latter method which was adopted. When calibrating a tube, the value of OD was variable, but since the value of $1 + \text{OD}^2/3r^2$ never exceeded 1·03, the equivalent length $\frac{1}{2}$ OD has been assumed as correct, the discrepancy being beyond the degree of accuracy desired. This approximation does not, however, suffice for determinations of cross-section. In such cases the complete formula has been employed.

The fact that in any determination of L and W the mercury column may not completely occupy the interior of the tube is not a serious disturbing factor, since, if L^2/W is accurately known for any one portion of the tube, its value for any other portion may be estimated from the calibration data.

Let L_1, L_2, S_1, S_2, W_1, and W_2 represent the relative lengths, mean cross-sections, and masses of mercury occupying two different portions of the same tube. Then

$$\frac{L_1^2/W_1}{L_2^2/W_2} = \frac{L_1 S_2}{L_2 S_1} = \frac{L_1 (n_1 + a_1 + b_1 + \ldots)(\Sigma W_2/\lambda_2 + a_2 W_2/\lambda_{a_2} + \ldots)}{L_2 (n_2 + a_2 + b_2 + \ldots)(\Sigma W_1/\lambda_1 + a_1 W_1/\lambda_{a_1} + \ldots)},$$

where the values of n_1, n_2, a_1, a_2, &c., are obtainable from the calibration data. The ratio S_2/S_1 is, however, more conveniently obtained from the calibration curve.

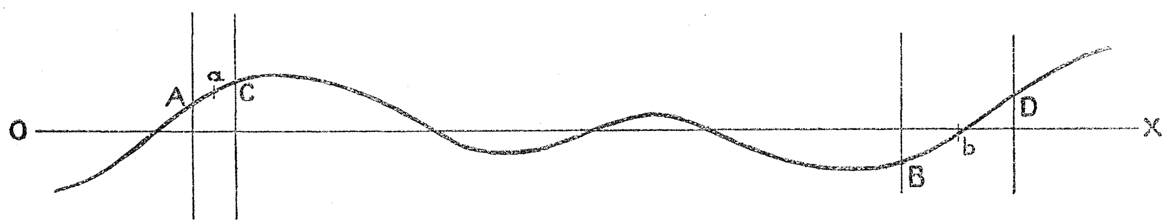

Fig. 3.

Let the limits AB (fig. 3) represent that portion of the tube employed for a mercury standard, and let OX be the mean line of the curve AB. The ordinate value of OX is the mean value of λ, the thread length employed in the calibration, and may be obtained by the use of a planimeter or by calculation of the mean λ. Let L_1 denote the length of the standard portion AB as measured along the axis, and S_1 be its mean cross-section. Let the limits of the mercury column employed to obtain a value of L_2^2/W_2 be denoted by C and D. The positions of C and D will vary slightly with the different measurements, but will always closely approximate to the limits A and B respectively. Also let L_2 denote the length CD as measured along the axis, and S_2 be the mean cross-section of this length.

Then the length L_1 is obtained by the algebraic addition of the two short lengths AC and BD to L_2. Let these short lengths be denoted by l' and l''. The mean cross-sections of these in terms of S_1 may be obtained from the positions of their mid-points a and b relatively to the zero line. Let them be written xS_1 and yS_1 respectively, where x and y are dependent on the ordinates of a and b. In consequence

$$S_1 = (L_2S_2 + l'xS_1 + l''yS_1)/(L_2 + l' + l'');$$

that is

$$S_1/S_2 = L_2/\{L_2 + l'(1 - x) + l''(1 - y)\},$$

so that

$$\frac{L_1^2}{W_1} \Big/ \frac{L_2^2}{W_2} = \frac{L_1}{L_2^2}\{L_2 + l'(1 - x) + l''(1 - y)\}.$$

From this expression it is seen that the length and position of L_2 is best chosen so that l' shall be nearly equal to l''; also, if x and y be of opposite signs, l' and l'' should be of the same sign, and *vice versâ*.

In general, L_1 is so nearly equal to L_2 that the expression $l'(1 - x) + l''(1 + y)$ is very small, and approximate values only of x and y are required.

Condition of Axis of Tube.

A matter of some importance is the curvature of the axis of the tube in different parts of the standard. The definition of the international ohm assumes the axis to be perfectly straight, but in practice it can only be approximately so. The cross-section varies considerably, and it is only reasonable to suppose that the axis (*i.e.*, the line joining the central points of the cross-sections) should also vary from the straight line. That the curvature in any one portion of the tube is not great is obvious by inspection, but there is no certainty from inspection that it may not considerably affect the resistance of a mercury column.

As an example, consider an axis of undulating form, and, for simplicity, let it be divided up into a number of arcs of equal curvature. Then for one of these, say AB, the length measured for the determination of L^2/W will be the straight line AB, in consequence of which L is too small, and therefore the resistance of the mercury column greater than that calculated. The true resistance of the portion AB may, however, be evaluated in the following manner.

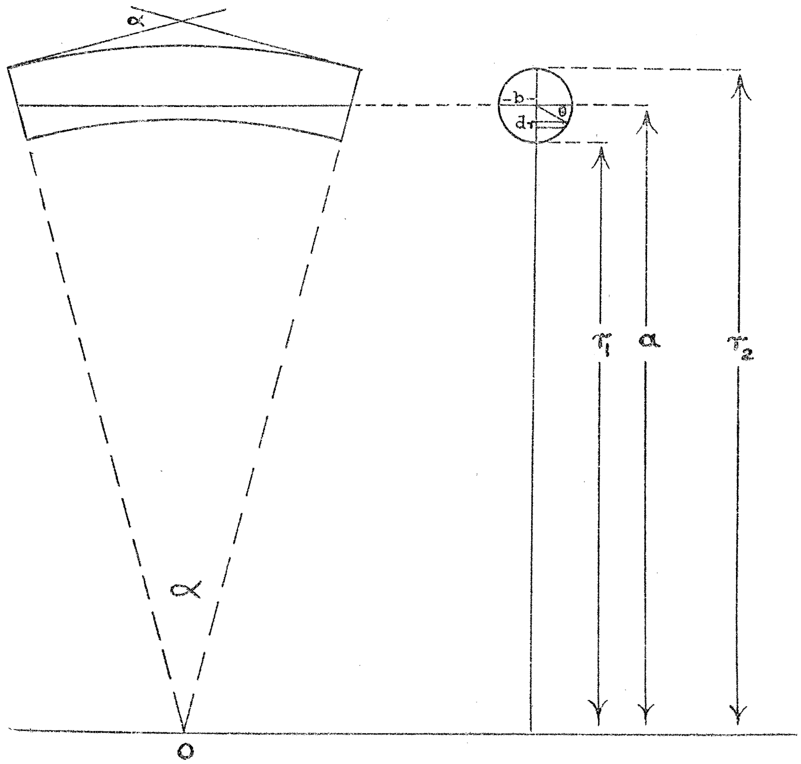

Fig. 4.

If we assume the curved portion to be of uniform cross-section, then the curvature will be different at different parts of the surface, and the surface may be imagined to be described by the motion of radii of different lengths, all parallel, and all emanating from a straight line. In fig. 4 the radii vary in length from r_1 to r_2, the mean radius of curvature being a. Let the angular motion of the radii be α. Then, if r be any one of the radii, the length of the corresponding element of the tube is $r\alpha$, and the cross-section of the element (see the figure) is $dr\sqrt{(r-r_1)(r_2-r)}$. Hence, if ρ be the specific resistance of mercury, and R the resistance of the column under consideration,

$$\frac{1}{\mathrm{R}} = \frac{1}{\rho}\int\sqrt{(r-r_1)(r_2-r)}\,\frac{dr}{r\alpha}.$$

If b be the radius of the cross-section of the tube, and $x = b\sin\theta$, $\mathrm{R}^{-1} = (\rho\alpha)^{-1}\int b^2\cos^2\theta\,d\theta/(a+b\sin\theta)$. Inserting the limits 0 and 2π, and neglecting the powers of b/a higher than the square, we thus obtain $\mathrm{R} = \frac{\rho\alpha a}{\pi b^2}\left(1 - \frac{b^2}{4a^2}\right)$, which is but slightly different from the approximate value $\rho\alpha a/\pi b^2$ calculated on the assumption that the elements were all of a mean length $a\alpha$. In practice, the length of the mercury column is assumed as equal to the straight line AB; also, since the volume of mercury is $a\alpha\pi b^2$, the cross-section is taken as $a\alpha\pi b^2/\mathrm{AB}$. Since $\mathrm{AB} = 2a\sin\frac{1}{2}\alpha$, the calculated resistance is $4\rho a\sin^2\frac{1}{2}\alpha/\pi b^2\alpha$. Thus the true and calculated resistances are in the proportion $\alpha^2 (1 - b^2/4a^2)/4 \sin^2 \frac{1}{2}\alpha$. To obtain an idea of the magnitude of this, let $\alpha = 1°$, $b = 0.05$ centim., and $a = 40$ centims., under which conditions $AB = 0.7$ centim. approximately; then the value of the above expression is 1·000036. Thus, supposing the axis of the tube to be of an undulating character, such that the curvature is everywhere equal to 0·025 ($\frac{1}{40}$ centim.), and that points of inflexion occur on the assumed axis of the tube at equal distances of 0·7 centim., then the resistance of the mercury column will be greater than that calculated on the assumption of a straight axis by 0·0036 per cent. Under these conditions, the maximum angle made by the real and hypothetical axes is 0·5° only, and the maximum displacement of the real from the hypothetical axis 0·015 millim. The true axis, if drawn on paper, could not be distinguished by the unaided eye from a straight line, and when the refractive effects of the walls of a tube be taken into consideration, the problem of determining the condition of the axis by experimental methods seems impossible. Indeed, such irregularity cannot be allowed for, and since it must hold good for every tube in some small measure, the calculated resistance must always be too small. The extent to which the axis undulates in this way will probably be largely dependent on the uniformity of the cross-section; this is an additional reason therefore for a careful choice being made. Consider the section of a tube by a vertical plane containing its axis. Let the upper bounding line be straight, so that all variations in the diameter of cross-section are shown by undulations in the lower bounding line. Then, for a tube of average radius 0·5 millim., with a maximum variation in cross-section of 4 per cent., the extreme displacement of the lower bounding line is 0·02 millim. The maximum displacement of the axis is therefore 0·01 millim. only.

Some criterion of the irregularity is afforded by the final results of the resistance measurements. If the probable error of all the determinations is not greater than 0·001 per cent., then the calculated and observed resistances should not differ by more than this. The greatest difference actually observed is 0·0036 per cent. (Table IX., Method I., Tubes V and Y). The axes, therefore, appear either to approximate very closely to straight lines, or to vary very much to the same extent.

It is evident from the foregoing that, when lengths are measured, every precaution must be taken to avoid bending the tubes. A slight curvature when observing the resistance is, however, of no consequence, as illustrated by the term $1 - b^2/4a^2$ being so nearly equal to unity.

Details of Operations.

The tubes employed were eleven in number. Eight of these were of Jena 16''' glass, obtained from Messrs. Schott and Co., while the remaining three were of "Verre dur," and procured from M. Baudin.

These eleven tubes were selected from a large number—sixty to eighty in all—the results of a rough calibration with a 5-centim. thread determining the choice.

Variations in cross-section as great as 10 per cent. were common, and it was a matter of some difficulty to select a dozen tubes all of which should have a small sectional variation.

The preliminary choice made, a long mercury column was introduced and its length and mass determined. A first approximation to the length for one international ohm was thus obtained. It was desirable that the shortest tube should be at least 60 centims. long, or the difficulty of accurately determining the mass of the column would be considerable; on the other hand, lengths greater than a metre would prove inconvenient, as they could not be readily measured.

Calibration.

As a preparation for calibration and other operations the tubes were cleaned by passing through them hot water, soda solution water, nitric acid, water, distilled alcohol, and twice distilled ether in the order named. In many cases a thin cotton covered wire, having attached to it a small plug of cotton wool, was passed through the tubes in a damp condition. This served to remove from the walls any adhering particles of solid matter. A rapid flow of the cleansing liquids through the tubes was produced with the aid of a filter pump, the latter also serving for the aspiration of air in the final drying. In every case the air used for drying was purified by its passage first through caustic soda solution, then through strong sulphuric acid, passing afterwards over dry phosphorus pentoxide and through a glass bulb heated to a temperature a little over 100° C. The air was thus ensured being warm and dry, while at the same time the tube itself was heated to a temperature approximating to 80° C. On cooling, the mercury was introduced by slight exhaustion.

The calibration was effected by mercury threads 5 centims. long. The reason for this choice is in part practical and in part the result of the theoretical reasoning already given. For the linear measurements a simple comparator by Messrs. PYE, of Cambridge, was employed, and the arrangements were not such as to facilitate the measurement of shorter lengths. For the tubes employed, however, 5-centim. lengths were sufficiently short for the necessary degree of accuracy to be obtained. As an indicating mark, a short line at right angles to the axis was etched near one end of the tube.

The 5-centim. thread being introduced, the ends of the tube terminated in two calcium chloride bulbs with small orifices. These latter dried the entering air and so prevented the mercury from becoming moist. The lengths of the menisci, however, were not so constant as when the tube was open to the atmosphere, owing to the slight variations in pressure produced by the motion of the thread. The order of measurement was as follows:—

The thread length for different mean positions along the axis of the tube was first determined. These first mean positions were at 5-centim. intervals, and in the majority of cases were designated by the numbers 0, 5, 10, 15, . . . and multiples of 5. The variations in the mean cross-sections of these 5-centim. lengths was thus obtained and enabled μ' (1st series) to be evaluated. (See Table XVIII.)

Next, the thread lengths for the positions 1, 6, 11, 16, &c., centims. were measured, thus dividing the tube into a new series, the μ' of which should be approximately equal to that of the first. Similarly, the third, fourth, and fifth series were determined, the whole of the measurements occupying about six hours.

No correction for varying temperature was necessary, for the room in which the calibration was conducted was maintained at a constant temperature by a silver ammonium-chloride thermostat. The small variations due to the proximity of the observer were found to be too small to affect the final results.

With respect to the uniformity of the tubes, the following table, giving the values of the conical corrections for mercury standards constructed by different observers is of interest. The same method of calculation has not, however, been generally employed.

TABLE II.

Mercury standards constructed by	No. of standard.	Nominal value in ohms.	μ.
Lord RAYLEIGH, 1882	I.	1	1 ·00314
	II.	1	1 ·00007
	III.	1	1 ·00046
	IV.	·5	1 ·000838
Dr. GLAZEBROOK, 1888	I.	$\frac{1}{3}$	1 ·00002
	II.	1	1 ·00045
	III.	$\frac{1}{3}$	1 ·00032
	V.	·5	1 ·00016
	VI.	1	1 ·00045
	VIII.	1	1 ·60079
	IX.	·5	1 ·00010
Phys.-Tech. Reichsanstalt, 1890 and 1893. .	XI.	1	1 ·001878
	XIV.	1	1 ·00034$_1$
	106	·5	1 ·000169
	114	1	1 ·000021
	131	2	1 ·000168
National Physical Laboratory, 1903	M	1	1 ·000069
	P	1	1 ·000104
	T	1	1 ·000008
	U	1	1 ·000058
	V	1	1 ·000199
	W	1	1 ·000085
	X	1	1 ·000033
	Y	1	1 ·000123
	Z	1	1 ·000099
	G	1	1 ·000141
	S	1	1 ·000023

Determination of L *and* W.

The best approximate positions for the extremities of the mercury standard were decided from a preliminary evaluation of L^2/W in conjunction with the formula given on p. 63. Whenever possible, x and y were chosen so as to be equal and of opposite sign. This was conveniently accomplished by integrating the area contained by the whole curve, the extreme ordinates and an abscissa. A mean line representing a mean thread length was thus obtained, and the positions of x and y chosen relatively to this. A second integration of the chosen length followed, necessitating in a few cases a slight adjustment of the favoured positions.

The tube having been cleaned and dried by the method already described, a mercury column was introduced. Its length and position were such that the extremities of the column coincided very nearly with the x and y limits. The ends of the tube were next capped with rubber tubing and glass plugs, wire being employed to bind the tubing and so prevent leakage. This latter was an important consideration, as the tubes were often immersed in water for several hours. The measurements of length were conducted by Mr. B. F. E. KEELING, who kindly submits the following report on the method employed and the degree of accuracy attained.

Determination of Length (Mr. B. F. E. KEELING).

The measurements of the mercury columns were made in a water-bath comparator furnished with two micrometer microscopes reading directly to 0·001 millim. The tube, with its mercury column, was closed at the ends with short lengths of india-rubber tubing closed with pieces of glass rod, and was then strapped into a groove in a stout brass bar. The clamps were sufficiently slack to allow of free contraction and expansion. This bar and a standard metre of known value and coefficient of expansion were supported side by side in the water bath, and were successively brought under the microscopes. The order of the readings was as follows:—Two readings were taken on the metre, then three on the lines of contact of the mercury and glass. The tube was then raised till the tops of the menisci were in focus, and three readings were taken. Next, the tube and metre were put out of adjustment and re-adjusted, and the readings repeated in the reverse order. The temperature of the water was observed four times, two thermometers being used.

Two metres were employed, one of nickel and one of brass, and both of these were compared with the standard metre of the National Physical Laboratory.

An example of the observations and necessary reductions follows:—

STANDARD Tube W.

Temperature.		Nickel metre. Micrometer readings. Left. 15·0 centims.	Nickel metre. Micrometer readings. Right. 91·0 centims.	Bases of menisci. Left.	Bases of menisci. Right.	Extremities of menisci. Left.	Extremities of menisci. Right.
° C.	° C.	*t.* *d.*	*t.* *d.*	*t.* *d.*	*t.* *d.*	*t.* *d.*	*t.* *d.*
17·62	17·63	11 98	7 81	—	—	—	—
	—	11 98	7 81	9 13	11 8	—	—
—		—	—	9 14	11 9	—	—
	—	—	—	9 13	11 8	10 98	9 51
—	—	—	—	—	—	10 97	9 51
17·63	17·63	—	—	—	—	10 98	9 49
17·62	17·64	—	—	—	—	10 92	9 45
—	—	—	—	—	—	10 92	9 46
—	—	—	—	9 25	11 21	10 92	9 45
—	—	—	—	9 25	11 19	—	—
—	—	12 7	7 92	9 26	11 19	—	—
17·62	17·64	12 6	7 92	—	—	—	—
17·62	17·64	12 2	7 86	9 19	11 14	10 95	9 48

Mean corrected temperature = 17°·55 C. Thus

	Left.	Right.
Length of menisci	= 176 divisions,	166 divisions,
	= 171μ,	160μ,
Equivalent columns	= 89μ,	83μ,
Total	172μ.	

Length of column = 76 centims. − 283 left divisions − 328 right divisions,
= 76 ,, − 274μ − 316μ,
= 75·9410 centims.,
Menisci = 0·0172 ,,

Correction for temperature of metre. . . } = + 0·0167 centims.,
Correction for error of metre . . . . . } = − 0·0023 ,,

Corrected length . . . = 75·9726 centims.

Distance from reference mark . . . . . } = 9·41 centims.

It will be noted that in nearly every instance the length of the meniscus is less than that calculated from the diameter of the tube. This is probably due to the fact that the tube was immersed in water at a temperature considerably lower than that of the room in which it was filled, and the consequent contraction of the mercury first affected the menisci.

No trouble was experienced from irregular refraction in the glass, except probably in the case of the tube P. A certain part of the tube had been selected and it was found impossible to get consistent values with different fillings, which, of course, occupied slightly different positions in the tube. Another portion of the tube was tried and no further trouble was experienced.

Determination of W.

The weights employed were by OERTLING, and were of brass, platinum, and aluminium. In the case of the platinum and aluminium weights the density of the material was assumed, that of platinum being taken as 21·5 and of aluminium 2·6. The density of the substance of the brass weights was determined by observations on the weights themselves, and a value of 8·421 chosen, this representing the mean of several very consistent determinations.

The standard weight employed was 100 grammes of bronze blanc from Sèvres. The value of this was given by the Bureau Int. Poids et Mesures in July, 1902, as 99·999143 grammes. Immediately previous to the work on mercury standards the laboratory weights were compared with this. A second bronze blanc weight was at the same time evaluated and despatched to the Board of Trade, where, through the kindness of Mr. CHANEY, it was compared with the International prototypes there. The value assigned to it by the Board of Trade was 100·00036 grammes. The value assigned to it at the Laboratory by direct comparison with the Sèvres weight was before despatching $100{\cdot}0005_2$ grammes, after receiving from the Board of Trade $100{\cdot}0005_6$ grammes, a maximum difference of 0·0002 per cent. only. Soon afterwards, close examination showed that the Sèvres weight was stained slightly by the silk lining of the box in which it was contained. Fearing that some chemical action was taking place, the weight was returned to Sèvres for re-polishing and re-evaluation. Its new value, as given by the Bureau, was 99·998824 grammes. The weight previously employed for comparisons was again evaluated, with the result that the value $100{\cdot}0005_3$ grammes was assigned to it. The small observed differences are clearly beyond the degree of accuracy contemplated with the mercury standards.

The laboratory weights by OERTLING were calibrated three times. First, as a preliminary to the weighings of mercury, secondly, when the work was half completed, and again at the conclusion of the observations. No differences greater than the errors of observation were recorded.

The balance was also by OERTLING, designed to carry a maximum load of 200 grammes and to respond to increments of a tenth of a milligramme. The sensitiveness was, however, increased beyond this, since small loads only were employed and stability could be ensured. In addition, the readings of the zero were rendered more accurate by avoiding parallax, and by optically magnifying the motion of the beam. These latter refinements were secured by fixing a plane mirror at the centre of the beam, the axis of oscillation of the mirror being in line with the knife edge of the central agate. A beam of light from an incandescent lamp first passed through a converging lens, and was then reflected from the mirror to a transparent scale, situated about 3 feet distant from the mirror. An adjustment of the position of the lens enabled a sharp image of the filament to be formed on the screen, and, since the axis of the optic beam lay in the horizontal plane containing the knife edge of the balance, small displacements of the zero were proportional to the disturbing forces. In this way the reading of the balance zero was a pleasure. In practice a change in zero of two scale divisions (1 scale div. = 1 millim.) corresponded to a hundredth of a milligramme.

After the length of the mercury column had been measured in the comparator bath, the capped tube was dried on the outside and left in a comparatively warm room for an hour or two hours. In some cases the tube was slightly warmed by a Bunsen flame held below and at a considerable distance from the tube. This was to ensure that the temperature of the mercury should be at least equal to that of the room, or otherwise deposition of moisture would produce trouble. The rubber caps were removed, and after examining the ends of the tube for moisture, the mercury was quickly run into a small weighed watch glass and removed to the balance case. The temperature of the latter was approximately constant, being situated in a constant-temperature room. In every case, however, the temperature was read together with the barometric pressure. Weighings were conducted on both pans of the balance, and the sensitivity frequently determined. The latter, however, was very constant indeed.

Calculations and Explanation of Table III. (pp. 74, 75)

It follows from the equations on p. 58 that the condition for a mercury column at 0° C. to have a resistance of one international ohm is that its length shall be equal to $106{\cdot}3^2\pi r^2\Delta/14{\cdot}4521\mu = 781{\cdot}872\pi r^2\Delta/\mu$, where μ is the conical correction of the column, r its mean radius, and Δ the density of mercury at 0° C. Since the calculated cross-section is an inverse function of the density of mercury, the length L is independent of the density. The value assumed for the latter in the calculations will be found to be afterwards eliminated, but a knowledge of the coefficient of cubical expansion for mercury is assumed. The coefficient given in LANDOLDT's Tables has been employed.

The approximate coefficients of expansion for Jena and Verre dur glass have been taken from HOVESTADT's treatise on Jena glass. The values given are 0·0000231 for Jena 16''' glass and 0·0000222 for Verre dur.

For each tube, L and the corresponding W were determined for several different, but approximately equal lengths, the extremities of the columns being situated in approximately the same positions. The measurements were made at some temperature other than 0° C., as will be seen by reference to Mr. KEELING's report. A correction was therefore necessary.

Let L_2 be the length at $t°$ C., corrected for all inaccuracies of the standard metre employed, the expansion of the same, and inclusive of the equivalent lengths of the menisci of the mercury column. Let W_2 be the corresponding weight of the mercury. Then the mean cross-section S of the tube at $t°$ C. is equal to $W_2/L_2\Delta(1-\gamma t)$, where Δ, as before, is the density of mercury at 0° C., and γ is the coefficient of cubical expansion.

If β be the coefficient of cubical expansion of the glass, then the mean cross-section S_2 at 0° C. is $W_2/L_2\Delta(1-\gamma t)(1-\frac{2}{3}\beta t)$, the glass being assumed as isotropic.

The mean cross-section S_1 of another length L_1, nearly equal to L_2, and occupying approximately the same position, may now be found with the aid of the formula on p. 63. For $S_1/S_2 = L_2/\{L_2 + l'(1-x) + l''(1-y)\}$.

On each standard tube an indicating mark was etched. When the observations for L_2 were made, the distance of this mark from the nearest extremity of the column was noted. The lengths l' and l'' were thus calculable. The standard length L_1 depended on the final grinding of the tubes; its exact position having been noted on the calibration curve, the mean line of the standard portion of the curve was obtained with an Amsler planimeter, the result afterwards being checked by calculation. The area bounded by that portion of the axis designated by l', the two ordinates passing from its extremities to the curve, and the portion of the curve thus isolated, was similarly found. In general, the mean line of this small area differed in position from that of L_1, but the ratio of the cross-sections was immediately determinable. Unit value being assigned to the cross-section of L_1, that of l' was found and is denoted in the formula by x. Similarly that of l'' is represented by y.

As an example of the calculation, the first measurements of the tube M (see also Table III.) are given.

In this example

$$L_2 = 59 \text{ centims. approximately.}$$

$$l' = -0\cdot25 \text{ centim.} \qquad x = 0\cdot9942.$$

$$l'' = -0\cdot46 \text{ centim.} \qquad y = 1\cdot0090.$$

Hence

$$(1-x) = 0\cdot0058 \quad \text{and} \quad (1-y) = -0\cdot0090,$$

and

$$L_2/\{L_2 + l'(1-x) + l''(1-y)\} = 59/(59 - 0\cdot00145 + 0\cdot00414).$$

That is

$$S_1/S_2 = 50/59 \cdot 0027.$$

And since $S_2 = \cdot 555374$ sq. millim., therefore $S_1 = \cdot 555349$ sq. millim.

It will be observed that it is the differences $1 - x$ and $1 - y$ which are most readily obtained from the calibration curves.

The determined values of l', $(1 - x)$, l'', and $(1 - y)$ are contained in the expression in column 15 of Table III. In order to find l' and l'', the distances of the mark from the ends of the tube are required. These distances may be obtained from the data in Table XVII. The estimated values of S_1 are tabulated in column 16 of Table III., and the length for one international ohm (obtained by means of the equation on p. 71) is given in column 17.

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TABLE III.—Giving all the Data (except that from which the Conical national Ohms of the Mercury Columns

Date.	Observation.	*t.* Temperature.	Distance of meniscus from mark.	*a.* Length of column to bases of menisci.	*b.* Lengths of menisci.	*c.* (*a*) + lengths equivalent to (*b*).	*d.* (*c*) corrected for temperature coefficient of metre.	L_2. (*d*) corrected for absolute error of metre.	W_2. Weight of column. All corrections made.	δ. Density of mercury at *t*° C.
		° C.	centims.	centims.	millims.	centims.		centims.	grammes.	
29.12.02	M 1	9·45	2·27	59·2166	·307 ·307	59·2326	59·2431	59·2416	4·46607	13·57227
2.1.03	2	8·44	2·97	58·7268	·146 ·126	58·7409	58·7502	58·7487	4·42912	13·57474
14.1.03	3	12·41	3·67	58·8642	·114 ·152	58·8779	58·8916	58·8901	4·43686	13·56497
16.1.03	4	8·13	3·27	58·5387	·123 ·172	58·5541	58·5630	58·5615	4·41518	13·57549
4.2.03	P 1	14·82	9·50	63·4717	·204 ·177	63·4920	63·5097	63·5080	5·14929	13·55909
6.2.03	2	15·60	9·80	63·4915	·172 ·181	63·5100	63·5289	63·5272	5·14999	13·55715
8.2.03	3	15·75	9·91	63·7952	·165 ·175	63·8131	63·8320	63·8303	5·17431	13·55681
9.2.03	4	16·53	9·64	63·2167	·194 ·176	63·2363	63·2557	63·2540	5·12745	13·55546
4.2.03	T 1	10·39	4·30	58·8279	·235 ·135	58·8475	58·8590	58·8574	4·34232	13·56994
6.2.03	2	11·96	5·30	58·8787	·181 ·145	58·8956	58·9088	58·9072	4·34494	13·56610
8.2.03	3	11·58	5·77	58·7174	·195 ·134	58·7347	58·7475	58·7459	4·33325	13·56703
5.5.03	U 1	16·20	7·05	62·0915	·123 ·169	62·1067	62·1193	62·1175	4·92309	13·55571
7.5.03	2	17·11	7·21	62·1087	·120 ·146	62·1224	62·1357	62·1339	4·92335	13·55348
8.5 03	3	17·32	7·01	62·1131	·123 ·153	62·1274	62·1409	62·1391	4·92405	13·55297
23.5.03	4	19·12	6·89	62·0025	·051 ·106	62·0105	62·0254	62·0236	4·91347	13·54856
6.5.03	V 1	16·94	3·82	73·5632	·147 ·132	73·5776	73·5932	73·5910	6·89676	13·55390
7.5.03	2	17·53	3·45	73·4510	·204 ·210	73·4730	73·4892	73·4870	6·88701	13·55245
8.5.03	3	17·34	3·73	75·5096	·127 ·148	73·5237	73·5397	73·5375	6·89150	13·55292
12.5.03	4	14·69	3·61	73·3285	·134 ·139	73·3425	73·3560	73·3538	6·87744	13·55941
4.5.03	W 1	17·13	8·90	75·9514	·114 ·171	75·9661	75·9824	75·9801	7·36436	13·55343
5.5.03	2	17·55	9·41	75·9410	·171 ·160	75·9582	75·9749	75·9726	7·36327	13·55240
6.5.03	3	17·04	9·23	75·7866	·105 ·192	75·8020	75·8182	75·8159	7·34870	13·55363
9.5.03	4	15·95	9·19	75·8199	·125 ·143	75·8336	75·8488	75·8465	7·35291	13·55632
21.5.03	5	16·40	9·30	75·9470	·112 ·151	75·9605	75·9759	75·9736	7·36477	13·55522
7.5.03	X 1	17·54	11·08	65·7205	·135 ·152	65·7354	65·7499	65·7479	5·50287	13·55243
8.5.03	2	17·25	11·22	65·5933	·159 ·161	65·6100	65·6242	65·6222	5·49295	13·55314
12.5.03	3	14·70	11·29	65·5347	·103 ·138	65·5470	65·5591	65·5571	5·48993	13·55939
20.5.03	4	15·85	11·31	65·6577	·074 ·197	65·6719	65·6849	65·6829	5·49939	13·55657
13.5.03	Y 1	14·94	12·18	62·3860	·143 ·120	62·3995	62·4112	62·4094	4·95594	13·55880
15.5.03	2	16·43	12·12	62·0717	·110 ·139	62·0845	62·0973	62·0955	4·93022	13·55515
15.5.03	3	16·06	12·05	62·3997	·108 ·160	62·4136	62·4264	62·4246	4·95620	13·55605
18.5.03	4	14·38	12·60	62·1772	·143 ·141	62·1919	62·2031	62·2013	4·94018	13·56017
9.5.03	Z 1	14·63	12·95	68·5622	·159 ·151	68·5783	68·5909	68·5889	5·99546	13·55956
12.5.03	2	14·70	13·08	68·3636	·155 ·160	68·3799	68·3925	68·3905	5·97786	13·55938
13.5.03	3	14·75	13·22	68·3827	·137 ·134	68·3967	68·4093	68·4073	5·97927	13·55926
14.5.03	4	15·07	13·01	68·3132	·096 ·128	68·3247	68·3376	68·3356	5·97259	13·55848
21.5.03	G 1	17·53	17·63	116·2993	·247 ·177	116·3215	116·3598	116·3568	17·28330	13·55245
21.5.03	2	17·60	17·55	116·2546	·192 ·141	116·2717	116·3102	116·3072	17·27662	13·55228
22.5.03	3	17·14	17·64	116·0737	·155 ·188	116·0913	116·1288	116·1258	17·25001	13·55341
23.5.03	4	17·87	17·62	116·2662	·185 ·197	116·2859	116·3250	116·3220	17·27729	13·55162
25.5.03	5	18·20	17·53	116·3906	·208 ·247	116·4145	116·4543	116·4513	17·29644	13·55081
23.5.03	S 1	17·88	18·76	119·6226	·175 ·185	119·6411	119·6813	119·6782	18·23057	13·55159
23.5.03	2	18·08	18·27	119·4975	·156 ·256	119·5193	119·5600	119·5569	18·21012	13·55110
26.5.03	3	17·35	18·59	119·3777	·253 ·263	119·4053	119·4444	119·4413	18·19547	13·55290
26.5.03	4	17·79	18·54	119·0210	·172 ·191	119·0397	119·0797	119·0766	18·13979	13·55182
27.5.03	5	17·36	18·30	119·2662	·167 ·112	119·2805	119·3196	119·3165	18·17581	13·55287

Correction is Calculated) necessary to Evaluate the Resistance in Inter-Occupying the Various Tubes.

S_2 at $t°$. $\frac{W_2}{L_2 \delta}$.	S_2. Mean cross-section at 0° C.	L_1. Measured length of ground tube.	$l'(1-x)+l''(1-y)$.	S_1 at 0° C. $=\frac{S_2 L_2}{L_1+l'(1-x)+l''(1-y)}$.	$S_1 \times 781 \cdot 872 \times \Delta$.	Mean.	Difference from mean.
sq. millims.		centims.		sq. millims.	centims.		
·555452	·555374	59·0259	−(·25 × ·0058) + (·46 × ·0090)	·555349	59·0336	59·0332	− ·0004
·555377	·555308	—	+(·45 × ·0060) − (·73 × ·0082)	·555339	59·0326	—	+ ·0006
·555411	·555309	—	+(1·15 × ·0060) − (1·29 × ·0080)	·555341	59·0328	—	+ ·0004
·555368	·555301	—	+(·75 × ·0060) − (1·22 × ·0080)	·555350	59·0337	—	− ·0005
·597982	·597846	63·4981	+(·4 × ·012) − (·41 × ·007)	·597829	63·5493	63·5489	− ·0004
·597968	·597824	—	+(·1 × ·012) − (·13 × ·007)	·597821	63·5485	—	+ ·0004
·597954	·597809	—	+(·01 × ·012) − (·32 × ·007)	·597830	63·5494	—	− ·0005
·597997	·597847	—	+(·26 × ·012) − (·02 × ·007)	·597820	63·5484	—	+ ·0005
·543679	·543592	57·7249	−(·52 × ·0034) + (1·65 × ·0034)	·543556	57·7801	57·7805	+ ·0004
·543701	·543601	—	+(·48 × ·0034) + (·71 × ·0034)	·543563	57·7808	—	− ·0003
·543690	·543593	—	+(·95 × ·0034) + (·08 × ·0034)	·543560	57·7805	—	± ·0000
·584658	·584513	62·0731	−(·11 × ·0110) − (·06 × ·0151)	·584533	62·1360	62·1360	± ·0000
·584630	·584477	—	−(·27 × ·0110) − (·20 × ·0151)	·584531	62·1358	—	+ ·0002
·584686	·584531	—	−(·07 × ·0110) − (·00 × · —)	·584538	62·1365	—	− ·0005
·584707	·584536	—	+(·05 × ·0110) + (·01 × ·0151)	·584529	62·1356	—	+ ·0004
·691443	·691263	73·5000	−(·34 × ·0113) − (·25 × ·0086)	·691319	73·4874	73·4881	+ ·0007
·691516	·691329	—	+(·03 × ·0113) + (·02 × ·0086)	·691324	73·4879	—	+ ·0002
·691468	·691283	—	−(·25 × ·0113) − (·21 × ·0086)	·691327	73·4882	—	− ·0001
·691454	·691298	—	−(·13 × ·0113) − (·274 × ·0086)	·691334	73·4889	—	− ·0008
·715132	·714944	75·9210	+(·05 × ·0067) − (·11 × ·0082)	·714949	75·9992	75·9993	+ ·0001
·715151	·714959	—	−(·46 × ·0067) + (·41 × ·0082)	·714956	75·9999	—	− ·0006
·715146	·714959	—	−(·28 × ·0067) + (·38 × ·0082)	·714947	75·9990	—	+ ·0003
·715125	·714950	—	−(·24 × ·0067) + (·31 × ·0082)	·714941	75·9984	—	+ ·0009
·715138	·714958	—	−(·35 × ·0067) + (·29 × ·0082)	·714958	76·0001	—	− ·0008
·617576	·617409	65·6338	−(·30 × ·0125) − (·42 × ·0055)	·617466	65·6368	65·6372	+ ·0004
·617611	·617447	—	−(·16 × ·0125) − (·15 × ·0055)	·617474	65·6376	—	− ·0004
·617599	·617459	—	−(·09 × ·0125) − (·02 × ·0055)	·617471	65·6373	—	− ·0001
·617607	·617456	—	−(·07 × ·0125) − (·12 × ·0055)	·617470	65·6372	—	± ·0000
·585672	·585538	62·1867	−(·20 × ·0203) − (·02 × ·0193)	·585580	62·2472	62·2471	− ·0001
·585736	·585588	—	−(·14 × ·0203) + (·23 × ·0193)	·585573	62·2465	—	+ ·0006
·585679	·585535	—	−(·07 × ·0203) − (·17 × ·0193)	·585579	62·2471	—	± ·0000
·585704	·585575	—	−(·62 × ·0203) + (·60 × ·0193)	·585584	62·2476	—	− ·0005
·644649	·644504	68·5199	−(·13 × ·0078) + (·19 × ·0083)	·644499	68·5103	68·5097	− ·0006
·644629	·644483	—	+(·00 × ·0078) − (·14 × ·0083)	·644494	68·5098	—	− ·0001
·644629	·644483	—	+(·14 × ·0078) − (·26 × ·0083)	·644493	68·5097	—	± ·0000
·644621	·644471	—	−(·07 × ·0078) − (·12 × ·0083)	·644486	68·5090	—	+ ·0007
1·096017	1·095733	116·507	−(·23 × ·024) − (·38 × ·020)	1·095854	116·4895	116·491$_2$	+ ·0017
1·096074	1·095789	—	−(·15 × ·024) − (·35 × ·020)	1·095886	116·4928	—	− ·0016
1·096004	1·095726	—	−(·24 × ·024) − (·62 × ·020)	1·095893	116 4935	—	− ·0023
1·096030	1·095740	—	−(·22 × ·024) − (·40 × ·020)	1·095863	116·4904	—	+ ·0008
1·096092	1·095797	—	−(·13 × ·024) − (·18 × ·020)	1·095860	116·4901	—	+ ·0011
1·124074	1·123777	119·472	+(·18 × ·0065) − (·03 × 0090)	1·123769	119·4569	119·454$_8$	− ·0021
1·123993	1·123692	—	−(·31 × ·0065) − (·40 × 0090)	1·123745	119·4543	—	+ ·0005
1·124026	1·123737	—	+(·01 × ·0065) + (·04 × 0090)	1·123733	119·4530	—	+ ·0018
1·124108	1·123812	—	−(·04 × ·0065) + (·36 × 0090)	1·123784	119·4574	—	− ·0026
1·123989	1·123700	—	−(·28 × ·0065) − (·13 × 0090)	1·123728	119·4525	—	+ ·0023

The standards S and G stand apart from the rest. These two were not measured by Mr. KEELING owing to the impossibility at that time of measuring any length greater than 110 centims. in the comparator bath. The method adopted with these was very similar to that employed by Mr. GLAZEBROOK in 1888.

The tubes were filled as before, but for length determinations, since liquid immersion was out of the question, the temperature was necessarily somewhat uncertain. The observations were made in the Electrical Standards Room, which, as before mentioned, is maintained at a constant temperature. Subject to the condition of a constant barometric pressure, the temperature of this room may be kept steady to 0·1° C. Here the tubes were placed in position, and every preparation made for the length determination, the comparator by PYE being employed. The mercury column having been adjusted, the menisci were focussed, and the arrangement left undisturbed over night, an open scale thermometer lying alongside the tube for temperature observations. Under favourable conditions, the temperature chart of this room for the night interval is a perfectly horizontal line, so that no greater error than ·002 per cent. (corresponding to 0·1° C.) was anticipated. Immediately on entering the room next morning, the temperature as indicated by the thermometer was read, the microscope slightly adjusted and the readings taken, the whole occupying about 1 minute. The standards of length were then substituted and the measurements rendered complete. The variations shown in Table III. are greater than expected, and the weight given to these two standards must necessarily be less than that given to the others.

Apart from the tubes S and G, the greatest difference from the mean is, in three cases only, greater than 0·001 per cent. Irrespective of sign, the mean difference is 0·0006 per cent., or six parts in a million.

The Cutting and Grinding of the Tubes.

The desired lengths and the positions of the extremities being known, the tubes had now to be cut.

To do this, a file cut was made in each case about 0·5 millim. distant from the point desired, so that, when broken, the standard portion was about a millimetre longer than requisite. This was advisable, since, on breaking, tiny splints of glass separated from the interior of the tube, making the cross-section there greater than before. The ends of the standard were next warmed and immersed in molten wax; on removing, the wax solidified, forming a solid plug about 3 centims. long at each end. Paraffin wax was at first tried, but melted too readily. A mixture of paraffin wax and beeswax in the proportion of three to one was eventually used and answered capitally.

For grinding the ends, a slide rest of one of the lathes of the laboratory was so adapted as to secure a small electric motor firmly in position on its upper surface. Originally it was desired that the ends of each tube should be perfectly plane surfaces at right angles to the axis, and the grinding arrangements were primarily intended to serve this purpose. Afterwards, however, a slightly convex surface was chosen, for reasons hereafter given. The figure (fig. 5) will best serve to explain the motions.

In the diagram (fig. 5) M is the small motor mounted on the slide rest, and, therefore, capable of travelling in the two directions *cd* and *ef* mutually at right angles. Other directions of motion inclined to *cd* were also possible by varying the angular reading at S. At the outset the reading at S was zero, and by attaching a wire pointer to the disc D the line of travel was determined. This is indicated by the line *ab* in the figure. At right angles to this *cd* was set off, and three short clamping pieces A, B, C were fixed so that their grooved portions were parallel to the

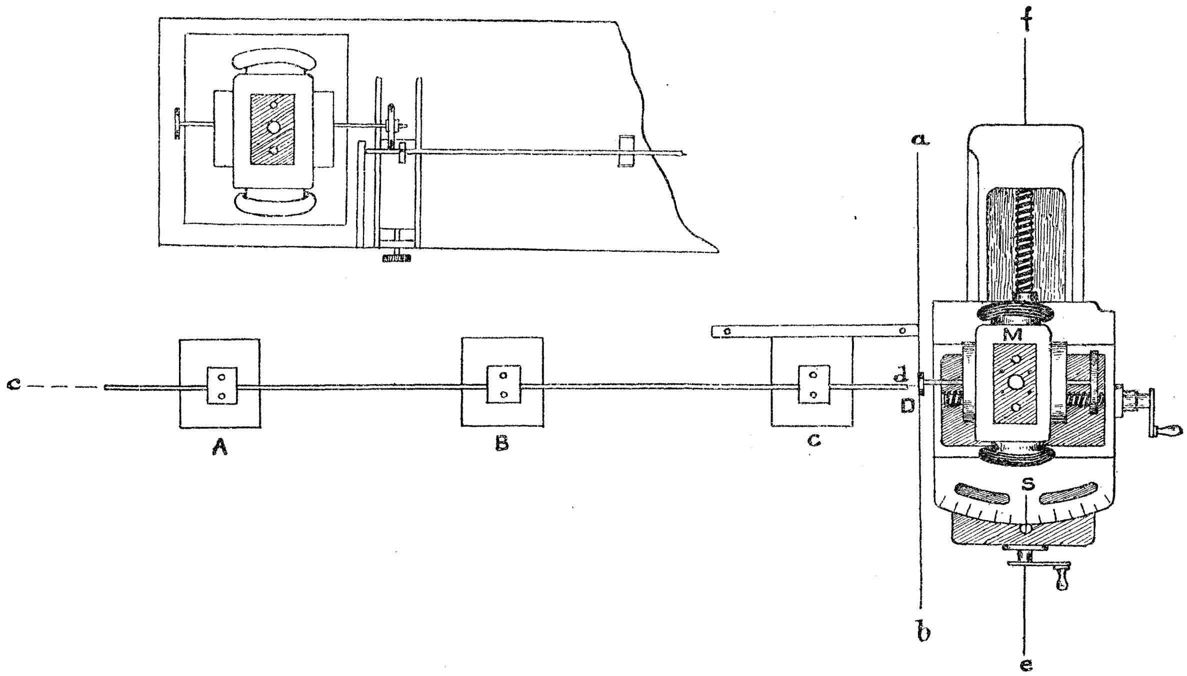

Fig. 5.

line *cd*. The plane in which the disc D rotated was now at right angles to the axis of the tube. D being a small disc about $1\frac{1}{2}$ inches in diameter, it was thought possible, once an end of a tube was approximately plane, to make it perfectly so, and, in addition, to lie at right angles to the axis. An attempt to attain this end by clamping the tube somewhat rigidly while D travelled to and fro failed. The failure is mainly attributable to the rigidity of the tube, for the vibration note was very high, and the risk of breaking considered too great. Instead, therefore, a slightly convex surface was ground.

At the commencement, a not too fine circle of emery cloth was attached to the disc and the speed adjusted to about 5 revolutions per second. The grooves in which the tube rested were lined with felt, the tube being quite free and capable of vibration as a whole. A gentle pressure of the tube against the disc commenced the grinding, a slight travelling motion of the motor being at the same time maintained. The wax prevented, to some extent, the detaching of tiny splints of glass from the interior of the tube, and also blocked the way to grains of emery, which are liable to scratch the inner skin of the tube and promote rupture. When sufficient grinding had resulted for both ends to present an even appearance, and the measured length tallied approximately with that desired, a slight convexity was given to the ends by deflecting the line of motion of the motor 1° to $1\frac{1}{2}°$, the tube being continually rotated. Also, owing to a slight splintering of the outer glass, a fine bevelled edge, inclined 20° to the ends, was ground. The final polish was produced by well-worn emery of the finest kind. Examination with a lens indicated that such a surface was very free from pittings and preferable to that obtained when the final polish was made with a copper or a brass disc. Throughout these grinding processes a lubricant of camphor and turpentine was used.

One of the methods adopted for measuring the resistance of the standards (see p. 81) requires the attachment of ebonite collars to the tubes. For this purpose three equidistant dimples were made at a distance of 1·3 centims. from each end. These were ground into the glass by a small emery wheel, the motor armature rotating slowly, and a gentle pressure maintained. The apparatus is depicted in the upper part of fig. 5.

With the measurement of each standard's length the mechanical constants of the tubes were completely known. Mr. KEELING determined these lengths (excepting those of G and S) in the comparator bath; his report is appended.

Determination of L_1 (Mr. B. F. E. KEELING).

The tubes, after they were cut and ground, were again strapped to the brass bar, holes being previously drilled in the bar so as to come directly under the ends of the tubes.

Two distinct measurements were made, the tube being turned through 180° between them. The microscopes were focussed directly on to the ends of the tubes, no contact pieces being employed. No difficulty in focussing was experienced.

Example of Measurements. Tube W.

(*a*.) Mark turned upwards.

Temperature.		Nickel metre. Left. 17·0 centims.	Nickel metre. Right. 92·9 centims.	Standard tube. Left.	Standard tube. Right.
° C.	° C.	*t*. *d*.	*t*. *d*.	*t*. *d*.	*t*. *d*.
16·8	16·8	—	—	10 26	9 86
—	—	—	—	10 24	9 87
—	—	—	—	10 27	9 86
—	—	9 46	10 84	—	—
—	—	—	—	10 25	9 84
—	—	—	—	10 26	9 85
—	—	—	—	10 27	9 86
16·8 ° C.		9 46	10 84	10 26	9 86

Length of column = 75·9 + 80 left divisions + 98 right divisions.
= $75·9 + 78\mu + 94\mu$.
= 75·9172 centims.

(*b*.) Mark turned downwards.

Temperature.		Nickel metre. Left. 17·0 centims.	Nickel metre. Right. 92·9 centims.	Standard tube. Left.	Standard tube. Right.
° C.	° C.	*t*. *d*.	*t*. *d*.	*t*. *d*.	*t*. *d*.
—	—	—	—	10 25	9 81
—	—	—	—	10 26	9 79
—	—	—	—	10 26	9 82
—	—	9 51	10 86	—	—
—	—	—	—	10 22	9 81
—	—	—	—	10 22	9 82
16·8	16·8	—	—	10 23	9 80
16·8 ° C.		9 51	10 86	10 24	9 81

Length of column = 75·9 + 73 left divisions + 105 right divisions = $75·9 + 71\mu + 101\mu$ = 75·9172 centims.

Mean of the two positions	=	75·9172 centims.
Correction for temperature of metre	= +	0·0159 ,,
,, ,, error of metre	= −	0·0023 ,,
Corrected length	=	75·9308 centims. at 16·8 ° C.
	=	75·9210 ,, ,, 0·0 ,,
Distance of end from reference mark	=	8·95 centims.

It is convenient at this point to give a summary of the data accumulated. In the following table is set forth :—

(1.) The length L of the standard at 0·0° C.

(2.) The theoretical length l for 1 international ohm, assuming the conical correction to be unity.

(3.) The value of the conical correction for each tube.

(4.) The theoretical length for 1 international ohm, the conical correction having been applied.

(5.) The calculated resistance at 0·0°C. of the mercury column occupying the tube.

TABLE IV.

Tube.	Glass.	L.	l. Theoretical length for 1 international ohm, assuming $\mu = 1$.	μ.	Theoretical length for 1 international ohm $= \frac{l}{\mu}$.	Calculated resistance of column in international ohms at 0° C.
		centims.	centims.		centims.	
M	Verre dur	59·0259	59·0332	1·000069	59·0291	$\cdot 99994_6$
P	Jena 16'''	63·4981	63·5489	1·000104	63·5423	$\cdot 99930_4$
T	,,	57·7249	57·7805	1·000008	57·7800	$\cdot 99904_6$
U	,,	62·0731	62·1360	1·000058	62·1324	$\cdot 99904_6$
V	,,	73·5000	73·4881	1·000199	73·4735	$1\cdot 00036_1$
W	,,	75·9210	75·9993	1·000085	75·9928	$\cdot 99905_5$
X	,,	65·6338	65·6372	1·000033	65·6351	$\cdot 99998_0$
Y	,,	62·1867	62·2471	1·000123	62·2393	$\cdot 99915_5$
Z	,,	68·5199	68·5097	1·000099	68·5029	$1\cdot 00024_8$
G	Verre dur	116·507	$116\cdot 491_2$	1·000141	$116\cdot 474_8$	$1\cdot 00027_5$
S	,,	119·472	$119\cdot 454_8$	1·000023	$119\cdot 452_3$	$1\cdot 00016_5$

The tubes M, V, X, Z, G and S were intended to have a resistance of approximately one ohm when fitted up in a certain manner. (See Erection Method I.) The remainder have a resistance approximating to one ohm when erected by Methods II. and III.

Erection of Standards for Resistance Measurements.

Three methods of erection have been employed. They are as follows :—

Method I.—The standard was so erected as to enable the resistance to be measured of that mercury column contained between the two *plane* termini of the tube.

Method II.—The erection was such as enabled the resistance to be measured between two points situated without the plane termini referred to above, thus introducing "end corrections" to the tube.

Method III.—The "end corrections" were necessarily introduced as in Method II. The resistance, however, was most conveniently measured by the Carey Foster Bridge. (Methods I. and II. do not admit of accurate measurement of the standards in this way.)

Before describing these methods it is necessary to give some explanation of :—

The "End" Effect of the Standard Tubes.

In the last column of Table IV., the calculation of the resistance of the standards is based on the assumption that the termini of the resisting column are planes at right angles to the axis of the tube and terminating the bore.

When the mercury tube terminates in two vessels, the dimensions of which are infinitely great compared with the diameter of the tube, and the whole is occupied with a conducting liquid, the resistance between two thick leads introduced into the vessels has been approximately calculated by Lord RAYLEIGH. The difference between this resistance and that first mentioned depends on (*a*) the liquid employed, (*b*) the terminal sections of the tube, (*c*) the dimensions of the end vessels and of the flange of the tube, and (*d*) the position of the leads; it is termed the "end correction." It is conveniently expressed as the resistance of two columns of the liquid, of lengths fr_1 and fr_2, and of respective radii r_1 and r_2, where f is a constant for similar dispositions of leads, vessels, &c., and r_1 and r_2 are the radii of the terminal sections of the standard tube.

Measurements of resistance when the standard is erected by Method I. (a description of which immediately follows) are independent of this "end correction." Those taken when the style of erection is that denoted by II. include it. The chief practical difference in the two cases is the great care required for one method of erection, and the simplicity of the other. As will afterwards be shown, Method of Erection I. is difficult, and the measurements are subject to a considerable probable error unless the greatest care be taken. For the purpose of checking wire standards of resistance this is unfortunate, since for such observations an easily reproducible and constant resistance is essential. Respecting Method II., the calculation of the theoretical resistance is less rigid than with Method I., but every important detail is easily reproduced.

Hence, for the first determination of resistance in international ohms, the standards have all been erected by Method I., while Method II. has been adopted for the reproduction of standards.

Erection of Standards. Method I.

When the standard portion was cut away from the calibrated tube, the connecting pieces were so marked that they and the standard could at any time be arranged in line in the original manner. Under such conditions, the cross-sections facing each other are similar in value, and the general contours of the areas the same. These end-connecting pieces were ground, and triple cuts made in them in a manner similar to the standard lengths already described. A connector was now desired, such that the three pieces of tube could be gripped together, with their axes in line, and without internal irregularity occurring at the junctions. For the purpose in view, these connectors could be made of almost any material; but ebonite was finally chosen as most suitable, owing to the small-grinding action of ebonite with glass.

The connector designed and adopted is in three parts, and is depicted with two tubes clamped together in fig. 6. Very little explanation is required. Through each

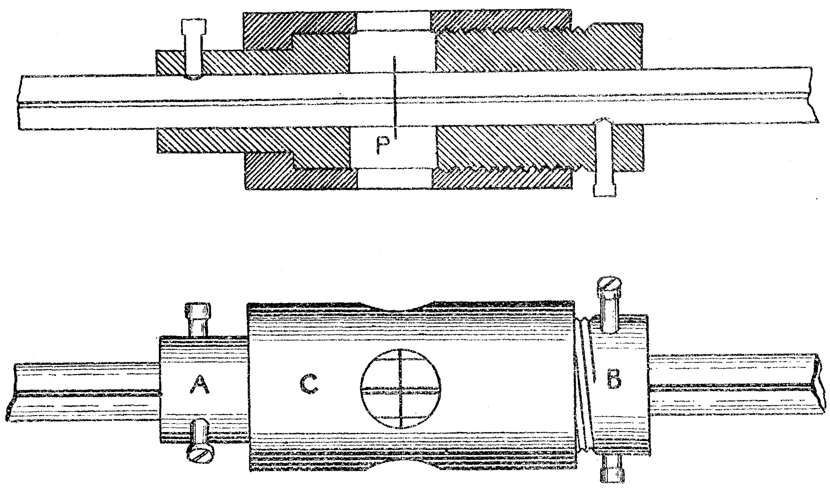

Fig. 6.

of the ebonite collars, A and B, three ebonite screws pass and grip the glass tubes by passing into the grooves previously ground. The collar A is flanged, while B is cut outwardly with a thread for about three-quarters of its length. The connecting piece C rotates freely about the flange of A, so that after the two tubes have been pulled together, the one may be rotated independently of the other. Four large circular holes are also drilled in C, enabling the junction of the tubes to be observed. P is a thin piece of platinum foil serving as a potential point for the mercury column.

For each standard a special pair of such connectors was made. The outer diameters of the tubes varying somewhat, a good fit in the collars A and B was otherwise impossible.

With a piece of perforated platinum foil in position as shown, and the tubes well gripped together, such a junction answered the following test. Direct exhaustion with a filter pump through the union reduced the internal pressure to a few centimetres of mercury, a column of mercury 72 centims. high being raised in a glass tube when the barometric height was 74 centims. On severing the connection between the filter pump and junction, the mercury gradually sank, so that at the end of an hour it stood at 60 centims. Possibly some leakage took place through the rubber connections.

The foil employed for the platinum potential points was 0·002 centim. in thickness, and was cut into strips 3 centims. long and about 3 millims. wide. In the centre of each strip, however, the width was greater than this, the foil being cut in the pattern of a disc of a diameter approximating to that of the outer wall of the tube. In the centre of this disc a hole corresponding to the bore of the tube was punched, the operations throughout being such as to prevent crinkling of the foil and fraying of the edges. When connecting two tubes in series, this platinum ring was laid on one of the ground faces, the two projecting strips bent over, and the ebonite collar B slid and screwed into position. The strips were thus gripped between the collar and tube, and the foil prevented from becoming excessively displaced. The corresponding portion of the connector—previously attached to the second tube—was now combined with A, and the junction rendered complete. In effecting this, neither tube is necessarily rotated about its axis, but when completed, some sight rotation is, in general, required to bring the two axes into line. An examination of the junction in two directions at right angles is rendered possible by the circular holes in the connector, although, as will presently be seen, a second examination is requisite. Once in their proper positions, the three tubes, comprising mercury standard and its two connecting pieces, must have no relative rotation about their common axis.

In practice, two other ebonite connectors joined the auxiliary tubes of the standard to others bent at right angles and intended for battery leads. These connectors are shown at *c* and *d* in fig. 7, and allow the leads *e* and *f* to freely rotate.

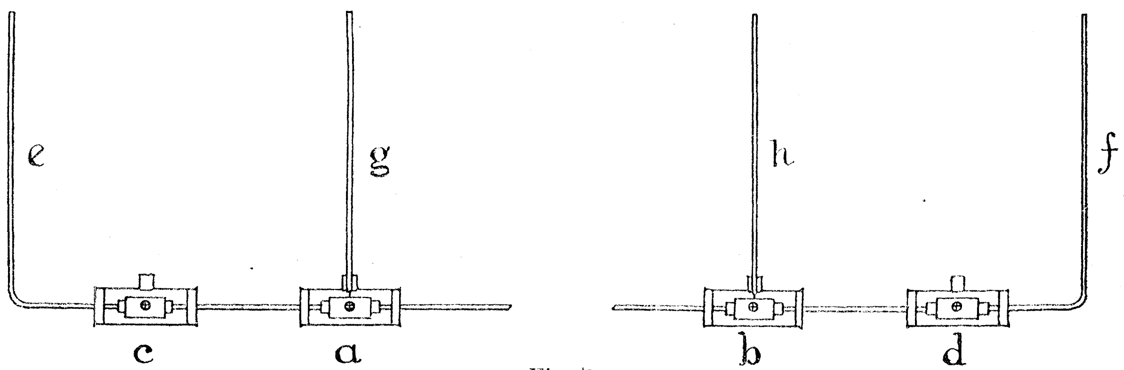

Fig. 7.

Smaller glass tubes with short platinum leads passed into *e* and *f*, and others similar to them served as conductors to the potential points of the standard. These are shown at *g* and *h*. Glass vessels a little more than 5 centims. long and about 2·5 centims. diameter covered the junctions. At *c* and *d* these vessels merely served to insulate the points; at *a* and *b* they were filled with mercury, the platinum leads dipping into the liquid.

The general scheme followed in fitting and filling the standard was as follows:— The tubes, and every part employed, were first carefully cleaned and dried, the parts having appropriate markings for their final positions. The junctions at *a* and *b* were next effected, the axes made to nearly coincide, and the four glass vessels slid over the connectors so as to rest on the standard tube. The junctions *c* and *d* were next made (*without* platinum foil), and afterwards the four vessels were passed into position and rendered as air tight as possible with rubber bungs. It should be observed that the purity of any mercury in the vessels at a and b is unimportant. When thus coupled together, the whole was rested on a framework of wood, to which—when filled—it was strapped, and placed in the ice-bath afterwards described. For filling, the lead e was turned downwards, and the tubular portions of the apparatus filled under reduced pressure with twice distilled mercury (see p. 105). The attendant conditions were the same as those which held in the determinations of L and W.

The adjustment of greatest importance had now to be made, viz., the ensuring of the correct relative positions of the standard and its connectors. How very important this is was not realised until many measurements of resistance had been made with very unsatisfactory results. However, by the close observance of the reflected light from the mercury columns, it is possible to secure great accuracy in this adjustment. Thus, if two cylindrical mirrors be placed with their axes parallel, parallel rays of light incident on both cylinders will produce two linear images. These will be in the same straight line for all positions of the observer if the axes are in exact line, but not otherwise. The various displacements will be equal to the projected displacements of the axes.

In the case of the mercury columns, the bright lines of reflected light acted in this way as a capital fine adjustment. They were made continuous by the rotation of the connecting tube, the junction being viewed first through one pair of holes in the collar C, and afterwards through the second pair. By successive approximations, and often slight adjustment of the screws securing the collars A and B in position, the continuity of the lines for both observations was secured. Examination with a lens greatly facilitated this. The standard portion and the connectors having originally been joined together, the refractive effects of the walls were equal and similar, and so were eliminated. Such an adjustment is, however, very tedious and may occupy as long as 20 minutes. Further remarks on the influence of this adjustment on the general accuracy of the measurements are made on p. 93.

The tube and accessories were now strapped in position on the wooden framework. Mercury was then poured into a and b, the platinum wires of the leads ignited, and these latter, which passed through small rubber bungs, pressed into position. The insulation of external junctions was made good by applying first a thin layer of hot paraffin wax, and afterwards a coat of shellac varnish. The insulation tests with paraffin wax alone were unsatisfactory when prolonged immersion in ice became necessary. The shellac varnish, however, was completely satisfactory.

By the introduction of the platinum battery leads into e and f the equipment of the tube was rendered complete.

The experience of previous observers indicates that for standard purposes the melting-point of ice is by far the most convenient temperature. No knowledge of the temperature coefficient of mercury is then assumed, and errors in thermometry are avoided. Consequently all standard observations were made at 0° C.

The ice box was a double one, with cork dust as a lagging. A double lid was also fitted, and under ordinary conditions the ice did not necessitate renewal for several days. In practice, however, a little was added each morning, the water being drained away at the same time. Two wooden rails were fixed to the base of the inner box, and corresponding grooves made in the wooden blocks of the framework supporting the tube. The latter could therefore be moved in the direction of its length only. This was found to be very convenient.

The leads of the standard passed to insulated terminals screwed on to one half of the hollow cover of the box. From these terminals thick copper leads made the necessary connections to the Kelvin double bridge. In addition, other wires were stretched to four terminals, the latter being conveniently situated for the measurement of the standard's resistance by the potentiometer method.

Measurement of Resistance. (*a*) *Kelvin Double Bridge* (fig. 8).

The circuit being closed, the condition for the potential difference at the galvanometer terminals to be zero is that P shall be equal to

$$\left(\frac{Q}{1+Q/W}+\frac{d\beta}{\alpha+\beta+d}\right)\left\{\frac{R\,(1+S/W')}{S\,(1+R/W'')}\right\}-\frac{d\alpha}{\alpha+\beta+d}.$$

When the value of QR/S is approximately one ohm, it may be determined by the substitution of a standard coil for P. If so found, the probable error is diminished,

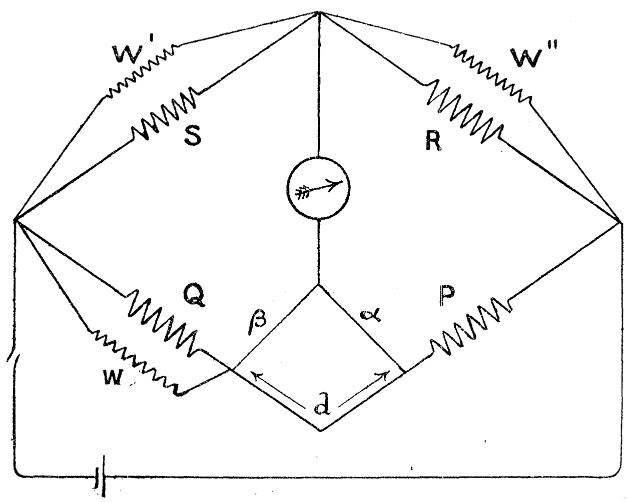

Fig. 8.

since accurate knowledge of the temperature coefficients of the coils Q, R, and S is unnecessary.

The substitution of a mercury standard for P causes the value of *d* to be much greater than usually attains. With standard coils, *d* has an average value of 0·00006 ohm, but with the mercury tubes it approximated to 0·15 ohm.

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In addition, a potential lead P_r (fig. 9) is thrown into series with R. [The value

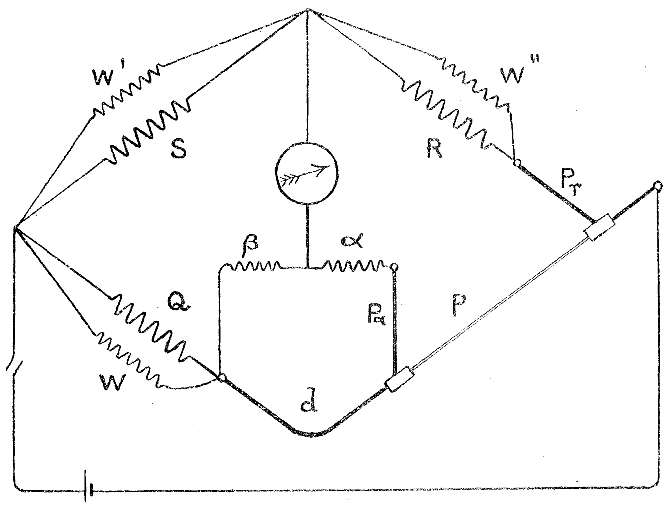

Fig. 9.

of P_r is of the order 0·016 ohm, and is conveniently measured to 1 per cent. of its value.] The complete expression for the value of P is therefore

$$\left(\frac{Q}{1+Q/W}+\frac{d\beta}{\alpha+\beta+d+P_\alpha}\right)\left\{\frac{R\,(1+R/W'')^{-1}+P_r}{S/(1+S/W')}-\frac{d\,(\alpha+P_\alpha)}{\alpha+\beta+d+P_\alpha}\right\}.$$

Owing to the magnitude of P_r, P_α, and d, and in order to reduce the corrections to small quantities, 1000-ohm coils were employed for R and S, while for α and β 100-ohm coils were used. Q was a standard-unit coil. All the coils were of manganin and had small temperature coefficients.

The ratio R/S was equal to 1·00002; that of α to β was 0·99998. Also, in no observation did $(1 + S/W')/(1 + Q/W)$ differ by as much as 0·2 per cent. from unity.

With this information, the above equation may be simplified to the form

$$P=\frac{QR\,(1+S/W')}{S\,(1+Q/W)\,(1+R/W'')}+\frac{P_r}{1000}-\tfrac{1}{2}\,d\left(\frac{10P_\alpha-P_r-\cdot 04}{1000}-\frac{1000}{W'}+\frac{1000}{W''}\right).$$

Such simplification depends, of course, on the values of the coils employed. It will presently be seen that P_r had a resistance never so great as 0·03 ohm; consequently, a value accurate to 1 per cent. was ample. The same remark respecting knowledge of value applies to P_α. The value of $(P_\alpha/100 - P_r/1000 - 0{\cdot}00004)$ was of the order $-$ 0·00015 ohm; that of d averaged 0·15 ohm, so that a value of d accurate to 1 per cent. was also sufficient to secure a final accuracy in the measurement of P of 0·0001 per cent. The method adopted for determining P_r, P_α, and d was, however, sufficiently good to ensure their estimated values being correct to 0·1 per cent.

With respect to the arrangements. The coils Q, R, and S were manganin coils of the British Association and made by WOLFF. They were immersed in oil contained in a double tank designed for the measurement of standards of resistance. This tank is shown in fig. 10 with the coils in position. M is a small electro-motor employed

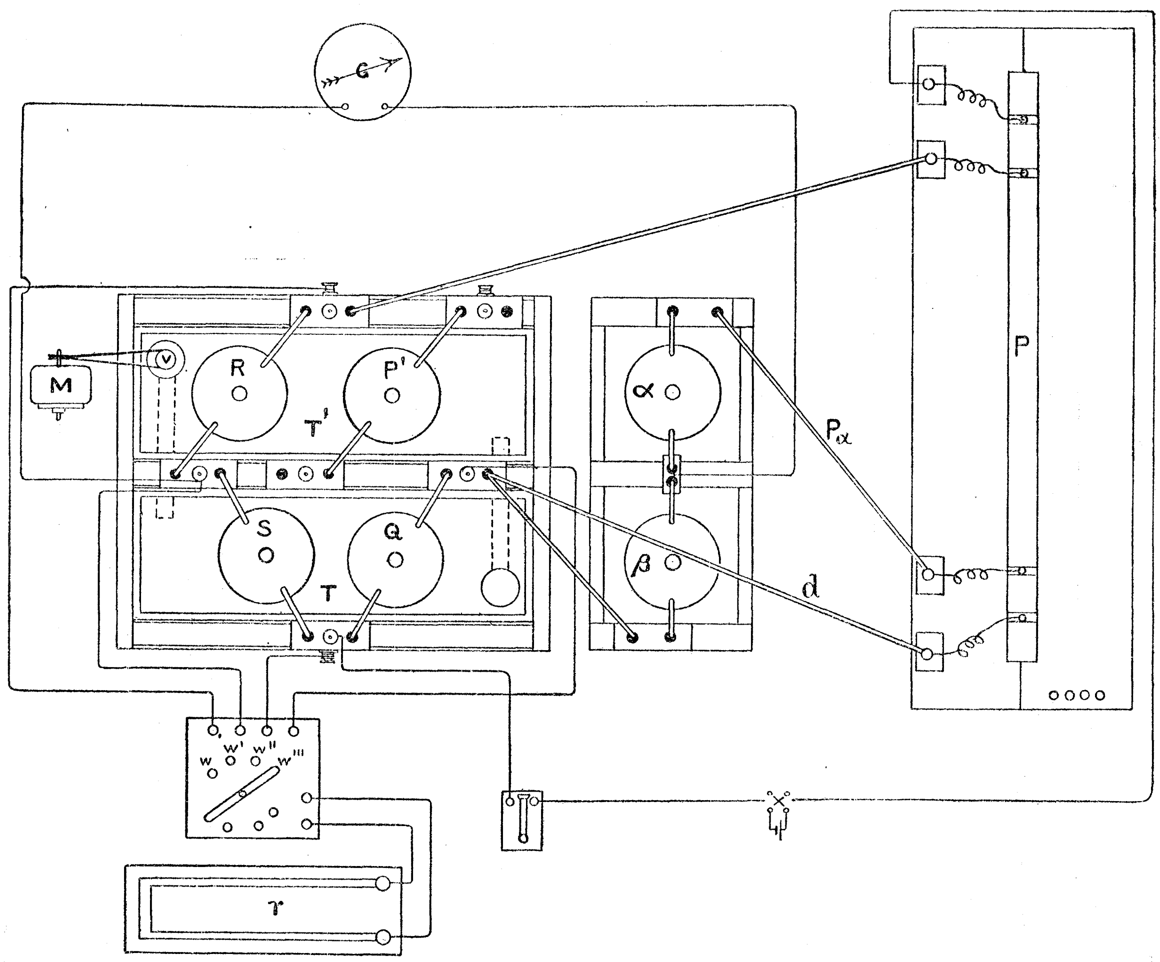

Fig. 10.

to drive the vane V. The stirring arrangements and communicating tubes are such that oil is pumped from the lower part of the tank T to the top of T′, and afterwards circulates from the bottom of T′ to the top of T. The whole apparatus is set up in a constant-temperature room, no difficulty being experienced in keeping the bath steady to 0·1° C.

The shunting of one of the coils Q, R, or S is effected by the turning head shown, the appropriate connections with the resistance box *r* being thus immediately made.

P′ is a standard manganin coil belonging to the British Association, and was used extensively throughout the comparisons. It is Wolff coil No. 1690, having a small temperature coefficient (+ 0·001 per cent. per rise in temperature of 1° C. for an average temperature of 17°·0 C.), and is known to have remained constant in value for several years. The value of 1690 has been obtained by means of a long series of observations with the platinum silver coils of the British Association. The unit in which 1690 is expressed is therefore assumed to be equal to 10^9 C.G.S. units of resistance (see pp. 114 and 115).

By introducing P′ into the circuit (the diagram shows it in a displaced position), the value of QR/S may be obtained.

When P′ is displaced, connections can be effected to the mercury standard. The leads of the latter were secured—as already described—to insulated terminals on the cover of the ice box. From three of these terminals $\frac{1}{4}$-inch copper leads passed to

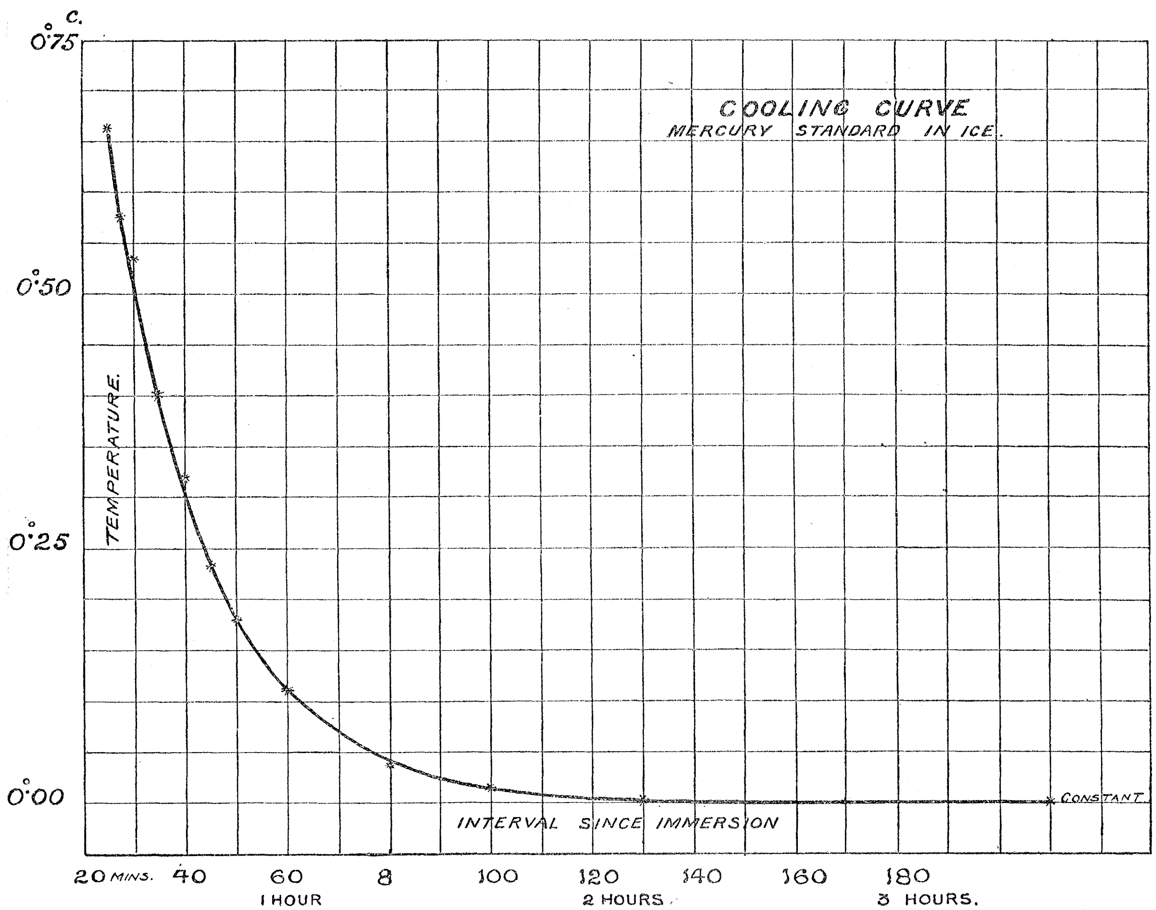

Fig. 11.

mercury cups in thick copper blocks, which latter formed part of the bridge. To the fourth terminal was secured one of the battery leads, and the connections thus rendered complete.

In practice, the tapping current employed was about 0·2 ampère, and with this a sensitivity corresponding to the evaluation of the resistance to 0·0001 per cent. was obtained. The galvanometer was one constructed at the laboratory with consequent pole magnets for the astatic system, as designed by Professor A. Broca, of the Ecole de Médecine, Paris. Its resistance was 500 ohms. The deflections were read telescopically at a distance of 4 metres, the scale divisions being transparent on a dark ground, with sunlight as an illuminant.

No measurements for standards purposes were made until the tube had been immersed in the ice shavings for at least 6 hours. Very intimate contact between the ice and glass was lacking owing to the melting of the ice in the vicinity of the tube and consequent formation of an ice wall some small distance away. The mercury, in consequence, attained the temperature of the ice but slowly. The cooling may be hastened by the application of pressure so as to produce better contact, and this was often done in the early stages of the immersion. The cooling curve shown in fig. 11 applies, however, to a normal case of cooling, the ice being undisturbed after the first 15 minutes. For the production of the curve, resistance measurements were made at equal intervals of time, and the temperature deduced from these observations. The first measurement of resistance was made 25 minutes after the tube had been immersed.

As an example of a final measurement at 0° C., the data for the standard X are given :—

June 18th, 1903.

Observations of Resistance. Mercury Standard X.

No. of observation, 1.

Insulation resistance = 20 megohms.

(*a*) Evaluation of QR/S.

				°C.
Q =	Wolff Coil No.	780,	temperature =	17·2
R =	,, ,,	740,	,, =	17·2
S =	,, ,,	2449,	,, =	17·2
P =	,, ,,	1690,	,, =	17·2

Balance was obtained when W = 9400 ohms.

The value of P at 17·2° C. = 1·000112 ohms.

Hence

$$QR/S = 1\cdot000112\,(1 + 1/9400) = 1\cdot000218 \text{ ohms.}$$

(*b*) Evaluation of Standard X.

Standard X substituted for 1690.

$$\alpha = 100\cdot013 \text{ ohms.}$$
$$\beta = 100\cdot015 \quad ,,$$

Balance was obtained when W = 7480 ohms.

Thus, since the total value of the corrections due to the leads has been shown to be small, the *approximate* value of X equals

$$1\cdot000218/(1 + 1/7480) = 1\cdot000084 \text{ ohms.}$$

(*c*) Evaluation of Leads.

The battery connection to the mercury standard was now broken, and the lead introduced at the junction of R and P_r. With these conditions holding,

$$QR/S = 1\cdot000218 \text{ ohms.}$$
$$P + P_r = \text{Standard X} + \text{Potential point } P_r.$$

Balance was obtained when W′ = 55070 ohms.

And since

$$P + P_r = \frac{QR/S\,(1 + S/W')}{(1 + Q/W)\,(1 + R/W'')} - \tfrac{1}{2}d\left(\frac{P_\alpha}{100} - 0\cdot00004\right),$$

we have $P + P_r = 1 \cdot 0184$ ohms approximately, so that $P_r = 0 \cdot 0184$ ohm, a value sufficiently accurate, easily evaluated, and determined immediately after the observation for P.

The coils α and β were now removed; the battery lead at the junction of R and P_r replaced to the junction of P and P_r, and the galvanometer connection made at dQ as shown in fig. 12. Under such

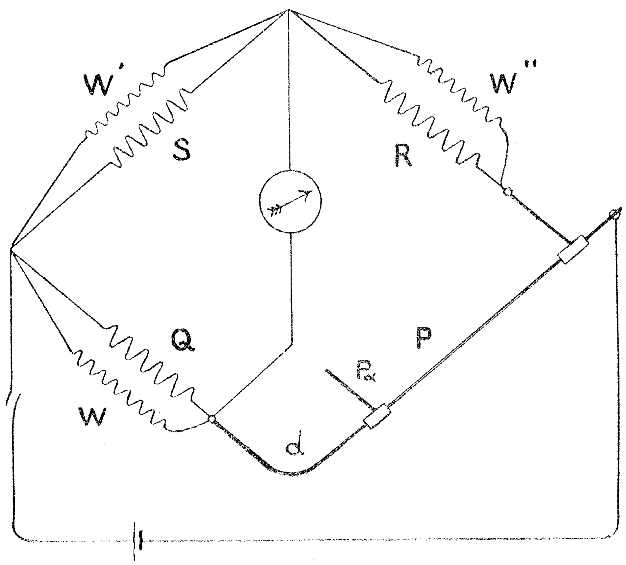

Fig. 12.

conditions $QR/S = 1 \cdot 000218 + P_r/1000 = 1 \cdot 000236$ ohms. Balance was obtained when $W' = 7055$ ohms. Hence $P + d = 1 \cdot 000236\ (1 + 1000/7055) = 1 \cdot 1419$ ohms approximately. Thus $d = 0 \cdot 142$ ohm, a value certainly correct to 1 per cent.

P_α was measured by substituting it for d. Under such circumstances $W' = 63690$ ohms to obtain a balance. Hence $P + P^\alpha = 1 \cdot 000236\ (1 + 1000/63690) = 1 \cdot 0160$ ohms approximately, and thus $P_\alpha = 0 \cdot 0160$ ohm.

P may now be accurately evaluated. Substituting the found values of P_r, P_α and d in the expression for P, we have

$$P = 1 \cdot 000218/1 \cdot 000134 + 0 \cdot 0184/1000 - \tfrac{1}{2} \times 0 \cdot 142\ (0 \cdot 0160/100 - 0 \cdot 0184/1000 - 0 \cdot 00004).$$

Thus resistance of standard tube $X = 1 \cdot 00009_5$ ohms.

Such was the general method adopted for the measurement of resistance by the Kelvin double bridge. As will be readily realised, the whole of the observations occupied but a very few minutes.

A correction has to be applied for the platinum foil introduced. The thickness of this was 0·002 centim., and although at the junction the cross-section was practically infinite, the limitation of the stream lines results in the resistance of the junction being equal to that of a mercury column of length 0·002 centim., and of the same cross-section as the united ends of the tubes. In consequence, the resistance as measured from the mid-points of the junctions will be greater than the calculated resistance in the ratio of $L_1 + 0{\cdot}002$ to L_1, where L_1 is the length of the tube in centimetres. In the case of X, L_1 is approximately 65·6 centims., so that the measured resistance must be reduced by 0·0030 per cent. Applying this correction, we have the resistance of the mercury column at 0° C. between two planes terminating the bore of the tube, equal to $1 \cdot 00006_5$ ohms, the unit of resistance being as previously specified.

Measurement of Resistance:—(*b*) *Potentiometer Method.*

Immediately after the determination by the double bridge, an observation by the potentiometer was made. It is convenient to describe this method now, before considering some of the possible errors.

The diagram (fig. 13) explains the connections.

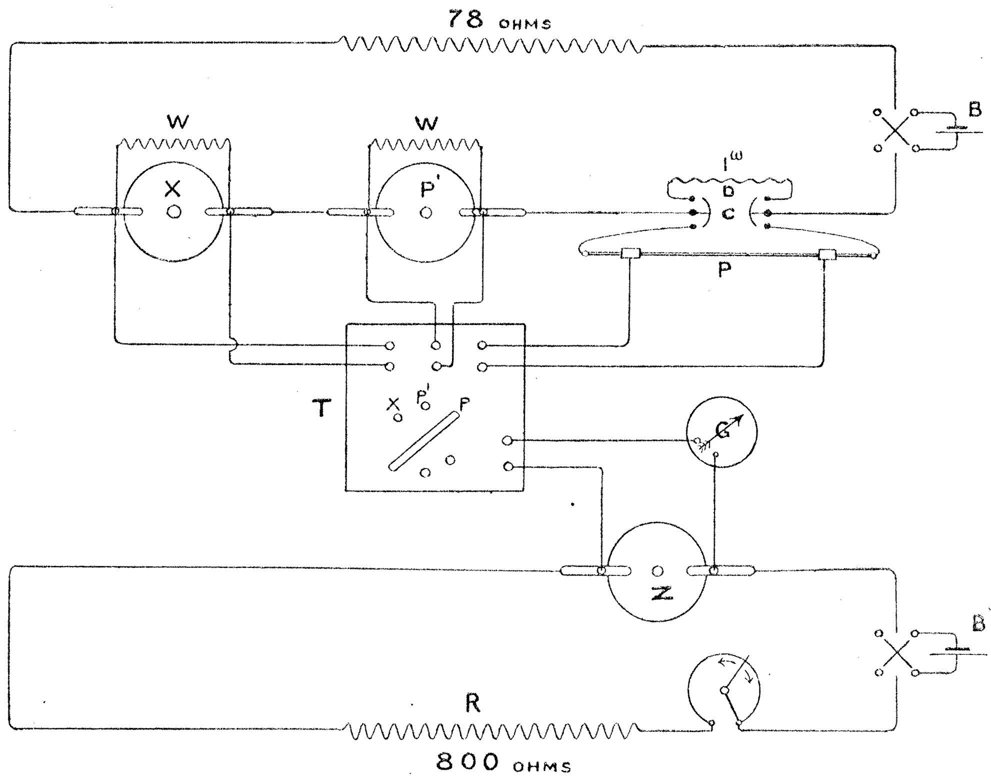

Fig. 13.

P is the mercury standard, P′ the comparison coil Wolff 1690, as before, X a standard 1-ohm coil of rather high value, and which when employed was evaluated by comparison with P. The additional resistance in this circuit was about 78 ohms, and the value of the current employed averaged 0·025 ampère.

In the second circuit, Z was a 10-ohm manganin coil, and the current employed averaged 0·0025 ampère. T was a turning head, arranged so that the potential points of either of the three resistances X, P′, and P could be quickly connected through the galvanometer to the potential points of Z. The galvanometer resistance was 5 ohms, the deflections being observed telescopically, as before, at a distance of

4 metres. The sensitiveness was even greater than with the double bridge, but owing to the nature of the method, the error was probably greater.

In practice, P was *not* initially introduced into the circuit. This was to avoid any warming of the mercury by the current. For it, D could be substituted by the switch C. The two currents were maintained until in a very steady state; P was then thrown into the circuit and a measurement made in the following manner:—

Balance between P and Z was first obtained by an adjustment of R. The resistance of X was greater than that of any of the mercury standards employed, and by an adjustment of the shunt W, a balance could now always be secured between X and Z. In some cases this was possible with P′. Afterwards the currents were reversed and balance again restored in the two cases, usually by some slight adjustment of W.

The value of P was thus known in terms of X (or, possibly, P′). In a similar manner, X was evaluated in terms of P′, the measurements being thus rendered complete. As before, a correction for the platinum-foil junctions has to be made. The thermal effects were usually equivalent to a difference in the shunted values of 0·006 per cent.

Each tube was thus measured by the Kelvin bridge and potentiometer at least three times. For each observation the tube was separated from its connectors, all the parts re-cleaned, dried, and fitted together again, as already described. Apart from the probable error of the observations, the consistency of the results is therefore entirely dependent on the state of the tube, the purity of the mercury employed, and the accurate adjustment of the connectors. Temperature variation is here ignored, since the repetition of this can certainly be secured.

The cleaning and drying of the parts, if always similarly conducted, should leave the tube always in the same state, that is, the thickness of air and liquid films should remain constant and be identical with those present when L and W were determined. The mode of ensuring that the axis of the tube is in line with the axes of the connectors has already been described. This method, however, was an afterthought. Originally, this adjustment was attempted when the tubes were empty. Approximate consistency in the resistance measurements was obtained, but variations of 0·003 per cent. were not uncommon, and occasionally differences as great as 0·01 per cent. resulted. The reason for these variations was not at first apparent. An accident to one of the connectors, however, led to some interesting measurements. One of these was slightly rotated by mistake, but the measurement was made without any attempt to rectify the displacement. The value of the resistance was 0·03 per cent. greater than usual, being equivalent to adding 0·2 millim. to the length of the tube. While in the ice box, the same connector was rotated through various angles, and measurements of the resistance made. The variation was as great as 0·05 per cent. The explanation, of course, lies in the deflection of the stream lines, the value of the resistance varying both with the displacement of the axes and with the position of the potential lead with respect to the section.

Thus in the figure (fig. 14), if OC and OD be the axes, and p the platinum lead dipping into the mercury, then, supposing the tube to be symmetrical with respect to the section, the potential of p will be the same as that of a, and therefore of A where A lies on the equipotential surface passing through a. If, now, the two tubes

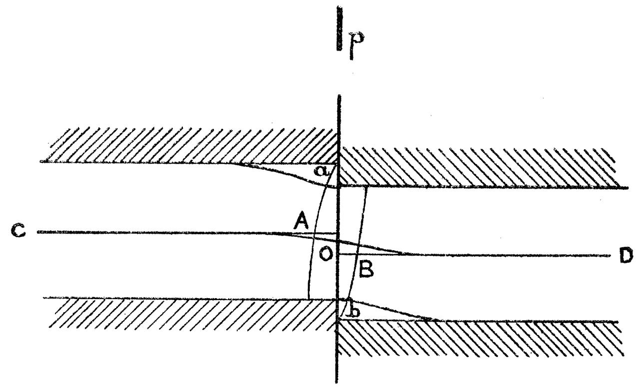

Fig. 14.

be rotated through 180°, the potential of p will, for the same reasons, be that of B. It is evident that if the displacement of the axes is considerable, the error is liable to be very great. This adjustment, therefore, is of the greatest importance, and much care and practice are required to attain success. The details of the adjustment have already been indicated.

The following table summarises the measurements of resistance for this mode of erecting the tubes. For comparison purposes, measurements by the Kelvin double bridge and the potentiometer are given together. Each horizontal line represents a separate filling of the tube :—

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TABLE V.—Observations : June and July, 1903.

Tube.		Resistance in ohms (Kelvin double bridge).	Resistance in ohms (potentiometer).	Mean value of resistance.	Difference from mean. Parts in 100,000.	
M (V.D.)	1	1·000055	1·000055	$1·00005_2$	$+0_3$	$+0_3$
	2	1·000046	1·000046		-0_6	-0_6
	3	1·000060	1·000052		$+0_8$	$\pm 0_0$
P (J. 16‴)	1	·999388	·999387	$·99938_4$	$+0_4$	$+0_3$
	2	·999379	·999395		-0_5	$+1_1$
	3	·999377	·999380		-0_7	-0_4
T (J. 16‴)	1	·999125	·999134	$·99912_1$	$+0_4$	$+1_3$
	2	·999120	·999120		-0_1	-0_1
	3	·999112	·999110		-0_9	-1_1
	4	·999123	·999124		$+0_2$	$+0_3$
U (J. 16‴)	1	·999119	·999120	$·99912_9$	-1_0	-0_9
	2	·999139	·999129		$+1_0$	$\pm 0_0$
	3	·999132	·999134		$+0_3$	$+0_5$
V (J. 16‴)	1	1·000436	1·000427	$1·00043_3$	$+0_3$	-0_6
	2	1·000430	1·000436		-0_3	$+0_3$
	3	1·000430	1·000440		-0_3	$+0_7$
W (J. 16‴)	1	·999148	·999135	$·99915_2$	-0_4	-1_7
	2	·999167	·999165		$+1_5$	$+1_3$
	3	·999146	·999152		-0_6	$\pm 0_0$
	4	·999148	·999151		-0_4	-0_1
X (J. 16‴)	1	1·000065	1·000071	$1·00006_3$	$+0_2$	$+0_8$
	2	1·000064	1·000070		$+0_1$	$+0_7$
	3	1·000055	1·000051		-0_8	-1_2
Y (J. 16‴)	1	·999271	·999268	$·99926_3$	$+0_8$	$+0_5$
	2	·999269	·999264		$+0_6$	$+0_1$
	3	·999254	·999254		-0_9	-0_9
Z (J. 16‴)	1	1·000332	1·000334	$1·00032_5$	$+0_7$	$+0_9$
	2	1·000317	1·000321		-0_8	-0_4
	3	1·000320	1·000324		-0_5	-0_1
G (V.D.)	1	1·000341	1·000342	$1·00034_2$	-0_1	$\pm 0_0$
	2	1·000337	1·000334		-0_5	-0_8
	3	1·000355	1·000346		$+1_3$	$+0_4$
S (V.D.)	1	1·000264	1·000262	$1·00025_4$	$+1_0$	$+0_8$
	2	1·000255	1·000251		$+0_1$	-0_3
	3	1·000244	1·000247		-1_0	-0_7

On the whole, the measurements made by the double bridge and the potentiometer are in good agreement. Where a considerable difference exists, more weight must be given to the Kelvin bridge observations, since, occasionally, balancing by the potentiometer method was troublesome as a consequence of slightly varying currents.

Also, the measurements resulting from different fillings differ from the mean value by a very small quantity. With few exceptions, this difference is less than 0·001 per cent.

The recorded measurements of the tube Z were made under slightly different conditions to those of the other tubes. Both glass connecting pieces of Z were fractured before any determination of the resistance had been made. The connecting pieces of X were substituted for them, since the terminal sections of the standard X are very similar to those of Z. The resulting resistance measurements were very consistent, the values obtained being as follows:—

Standard Z. — Resistance by double bridge.	(Connecting tubes of Standard X.) — Resistance by potentiometer.
ohms.	ohms.
1·000285	1·000299
1·000309	1·000305
1·000301	1·000296

As a check on these observations, the fractured connecting pieces of Z were re-ground and used as originally intended. The cross-sections of the ends of the tubes now brought into contact were but very slightly different. Possibly there existed some slight difference in the shape of the sections, but certainly the connecting pieces were preferable to those of X. The results of the measurements are those given in Table V. It will be observed that the values are greater than those above by about 0·003 per cent. The same weight cannot, however, be attached to these observations as to those of the other tubes.

Method II. of Erecting the Mercury Standards.

For the following method of erecting mercury tubes, the advantages claimed are:—

(1) Greater ease in manipulation and filling;
(2) Greater consistency in the results, and therefore a less probable error;
(3) Greater ease in the reproduction of resistance over short, and, more especially, long intervals of time.

The personal element is also introduced to a less extent, since in this second method the operations are much less exacting.

The disadvantage lies in the uncertainty of what is known as the "End Correction"; this renders the calculation of the theoretical resistance somewhat difficult. From a reproduction standpoint, however, the uncertainty is of no consequence.

The essential difference between this and the first method lies in the fact that the connecting tubes and standard are no longer in contact, but are separated by an interval of 3 centims. The glass connecting vessels are the same as before, the tubes passing through rubber bungs fitting tightly into the vessels. The action of mercury on rubber has to be rendered negligible, and, should the erection be a permanent one, contact with rubber is most inadvisable. Under such conditions the ends of the tubes may be ground similar to glass stoppers, and vessels constructed accordingly. Such permanent connections have been designed and will shortly be employed. The rubber bungs used were first of all boiled in a weak solution of caustic soda, and afterwards well sandpapered whilst wet. This operation disposed of much of the free sulphur present. After several such boilings in soda and in water, they were dried and immersed for some time in hot liquid paraffin wax. A final washing followed, and the bungs were dried and ready for use. The thin film of wax on the surface of the rubber effectually prevented any action between the latter and mercury. For example, one tube was filled and remained in the ice bath for 14 days, the resistance being measured daily. No difference whatever was detected in these measurements.

The arrangement adopted for the erection of the standard is shown in the figure (fig. 15).

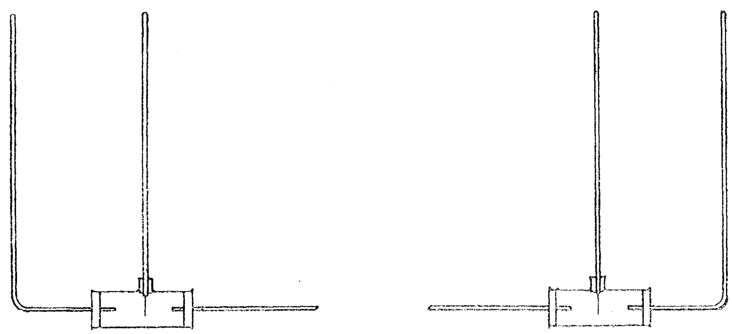

Fig. 15.

The theoretical resistance is now greater than before, being equal to that of the standard, plus the resistance of mercury between the ends of the tube and the potential points. Lord Rayleigh has shown* that when the stream lines diverge into an infinite volume of mercury, the flange of the tube being also considered as infinite in extent, that the value of f in the expressions fr_1 and fr_2 (see p. 81) is approximately 0·82. Thus under these conditions, an additional resistance is imposed on the tube which is equal to the resistance of two columns of mercury of lengths $0{\cdot}82r_1$ and $0{\cdot}82r_2$ and of respective cross-sections πr_1^2 and πr_2^2, where r_1 and r_2 are the respective radii of the terminal sections of the standard.

Such conditions, however, did not even approximately prevail in the measurements

* "Theory of Sound," § 307 and Appendix A.

made, nor is it likely that any theoretical computation (for the particular connecting vessels employed) would be so accurate as an experimental determination.

Throughout the experiments, the distance between the plane ends of the standard and the potential points was maintained constant. It was equal to 1·5 centims. The constancy of this distance was of some importance, since the resistance of a column of mercury 1 centim. long and of the cross-section of the connecting vessels is 0·000026 ohm (cross-section of connecting vessel = 4 sq. centims.). Now, although the value $f = 0{\cdot}82$ is from a theoretical standpoint a little too great for the conditions imposed, it is sufficiently near to the true value to indicate that the difference of the resistance measurements, when erection Methods I. and II. are employed, is of the order 1 per cent. for unit tubes. If, then, the resistance measurements in the two cases are accurate to 0·001 per cent. only, the value of f can only be calculated with an accuracy of 1 per cent. To attain greater accuracy, the following method was adopted :—

One of the tubes (H, Jena 16‴), originally intended for a mercury standard, became of little value owing to the development of a flaw near one end. Its cross-section was, however, accurately known, and the calibration curve had been plotted. At the point where the flaw had developed, the tube was broken, and the new end ground, so as to avoid irregularities. It was then filled with all the care pertaining to a standard, and its resistance measured by the Kelvin double bridge and the potentiometer, the mode of erection being necessarily Method II. The value so obtained was $0{\cdot}95109_2$ ohm. The length of the tube was 98·9 centims. and the mean cross-section 0·9792 sq. millim.

After removal from the ice bath and subsequent cleaning, the tube was cut into five lengths, these being respectively 10, 25, 22, 29, and 13 centims. The corresponding points of fracture were marked on the calibration curve and the cross-sections at these points calculated therefrom. The divided tube was then re-erected, four glass vessels serving to connect the five portions, and a distance of 3 centims. maintained between any two. Thus the total "end corrections" were now considerable, and, since the junctions were all similar and the cross-sections well known, a fairly accurate evaluation of f was possible. The resistance at 0·0° C. was now $0{\cdot}95450_3$ ohm, showing a difference of $0{\cdot}00341_1$ ohm from the first measurement.

The mean cross-section of the whole tube was, as above stated, equal to 0·9792 sq. millim.

The cross-sections and radii of the new ends introduced were :—

	Cross-section.	Radius.
	sq. millim.	millim.
(1)	0·9669	0·5548
(2)	0·9841	0·5597
(3)	0·9987	0·5638
(4)	0·9660	0·5545

Thus, since the resistance of a mercury column of length L and cross-section S is L/106·300 ohms, the constant f must be such that

$$\frac{2f}{106\cdot300}\left(\frac{0\cdot05548}{0\cdot009669}+\frac{0\cdot05597}{0\cdot009841}+\frac{0\cdot05638}{0\cdot009987}+\frac{0\cdot05545}{0\cdot009660}\right)=0\cdot003411_1,$$

whence $f = 0\cdot795$. We may now with some confidence apply this constant to obtain the end corrections of the various tubes. In Table VI. these corrections are evaluated and expressed as small lengths to be added to the standard length. The method of calculation is as follows:—

Let r be the mean radius of the standard, and r_1, r_2 the radii of the sections at the ends. Then the end corrections are equivalent to two mercury columns at 0° C. of lengths $0\cdot795r_1$ and $0\cdot795r_2$, and cross-sections πr_1^2 and πr_2^2. The lengths of two columns of uniform section πr^2 which have the same resistance as these are $0\cdot795r^2/r_1$ and $0\cdot795r^2/r_2$, and these are the two lengths to be added to the standard. The complete correction is conveniently written in the form $0\cdot795r^2(r_1+r_2)/r_1r_2$. Table VI. gives the necessary data for the calculations.

TABLE VI., in which the End Corrections are Evaluated.

Tube.	Mean cross-section at 0·0° C.	Mean value of r^2.	Cross-sections at— Marked end.	Cross-sections at— Unmarked end.	Marked end. r_1.	Unmarked end. r_2.	Value of $\cdot795r^2\left(\frac{r_1+r_2}{r_1r_2}\right)$.
	sq. millim.		sq. millim.	sq. millim.	millim.	millim.	
M (V.D.)	·5553	·17677	·5581	·5603	·4215	·4223	·666
P (J.)	·5978	·19029	·5939	·5939	·4348	·4348	·697
T (J.)	·5436	·17303	·5479	·5474	·4176	·4174	·659
U (J.)	·5845	·18605	·5772	·5936	·4286	·4347	·685
V (J).	·6913	·22005	·6834	·6979	·4664	·4713	·745
W (J.)	·7150	·22759	·7100	·7089	·4754	·4750	·761
X (J.)	·6175	·19652	·6251	·6137	·4460	·4420	·704
Y (J.)	·5856	·18640	·5739	·5739	·4274	·4274	·694
Z (J.)	·6445	·20515	·6500	·6493	·4549	·4546	·718
G (V.D.)	1·0959	·3489	1·0697	1·1179	·5835	·5965	·940
S (V.D.)	1·1237	·3578	1·1307	1·1139	·5999	·5954	·952

The values of the end corrections enable us to now calculate the resistance in international ohms of the standards as erected by Method II. Approximately the resistance is 0·1 per cent. greater than before.

TABLE VII.

Standard.	Length. L_1.	End correction $= l$.	Equivalent length $= L_1 + l$.	Method I. Theoretical length for 1 international ohm.	Method II. Theoretical resistance in international ohms.
	centims.	centim.	centims.		
M	59·0259	·0666	59·0925	59·0291	$1·00107_4$
P	63·4981	·0697	63·5678	63·5423	$1·00040_1$
T	57·7249	·0659	57·7908	57·7800	$1·00018_7$
U	62·0731	·0685	62·1416	62·1324	$1·00014_8$
V	73·5000	·0745	73·5745	73·4735	$1·00137_5$
W	75·9210	·0761	75·9971	75·9928	$1·00005_7$
X	65·6338	·0704	65·7042	65·6351	$1·00105_3$
Y	62·1867	·0694	62·2561	62·2393	$1·00027_0$
Z	68·5199	·0718	68·5917	68·5029	$1·00129_6$
G	116·507	·0940	116·601	116·475	1·00108
S	119·472	·0952	119·567	119·452	1·00096

The methods adopted for the measurement of resistance were the same as before in every respect. On examining the results set forth in the following table, greater consistency is, however, evident. This is undoubtedly due to the elimination of the ebonite connector. No greater difference from the mean than 0·0008 per cent. was observed.

These observations may be regarded as standard ones intended for the maintenance of the Laboratory's wire standards of resistance. The time occupied in the erections and measurements, however, was not more than one-third of that when Method I. was employed.

TABLE VIII.—Showing the Resistance of the Standard Tubes as Erected by Method II. Each Horizontal Line indicates a Separate Filling. Observations: July and September, 1904.

Tube.	Resistance in ohms (Kelvin double bridge).	Resistance in ohms (potentiometer).	Mean value of resistance.	Difference from mean. Parts in 100,000.	
M 1	1·001162	1·001167		-0_2	$+0_3$
2	1·001164	1·001162	$1·00116_4$	$\pm 0_0$	-0_2
3	1·001166	1·001165		$+0_2$	$+0_1$
P 1	1·000467	1·000468		-0_3	-0_2
2	1·000468	1·000472	$1·00047_0$	-0_2	$+0_2$
3	1·000472	1·000471		$+0_2$	$+0_1$
T 1	1·000277	1·000273		-0_1	-0_5
2	1·000281	1·000280	$1·00027_8$	$+0_3$	$+0_2$
3	1·000276	1·000281		-0_2	$+0_3$
U 1	1·000215	1·000213		-0_2	-0_4
2	1·000216	1·000216	$1·00021_7$	-0_1	-0_1
3	1·000222	1·000218		$+0_5$	$+0_1$
V 1	1·001462	1·001455		$\pm 0_0$	-0_7
2	1·001468	1·001462	$1·00146_2$	$+0_6$	$\pm 0_0$
3	1·001463	1·001465		$+0_1$	$+0_3$
W 1	1·000156	1·000161		$+0_3$	$+0_8$
2	1·000152	1·000151	1·000153	-0_1	-0_2
3	1·000146	1·000151		-0_7	-0_2
X 1	1·001147	1·001147		-0_4	-0_4
2	1·001154	1·001151	$1·00115_1$	$+0_3$	$\pm 0_0$
3	1·001155	1·001152		$+0_4$	$+0_1$
Y 1	1·000356	1·000354		$+0_6$	$+0_4$
2	1·000347	1·000350	$1·00035_0$	-0_3	$\pm 0_0$
3	1·000348	1·000346		-0_2	-0_4
Z 1	1·001387	1·001391		-0_2	$+0_2$
2	1·001389	1·001384	$1·00138_9$	$\pm 0_0$	-0_5
3	1·001392	1·001393		$+0_3$	$+0_4$
G 1	1·001130	1·001135		-0_5	$\pm 0_0$
2	1·001138	1·001135	$1·00113_5$	$+0_3$	$\pm 0_0$
3	1·001131	1·001132		-0_4	-0_3
S 1	$1·00105_3$	$1·00105_4$		-0_4	-0_3
2	$1·00105_5$	$1·00106_1$	$1·00105_7$	-0_2	$+0_4$
3	$1·00106_1$	$1·00105_7$		$+0_4$	$\pm 0_0$

It is convenient at this point to tabulate for each tube the difference

Observed resistance — theoretical resistance,

the former being expressed in terms of 10^9 C.G.S. units of resistance, while the latter are international ohms.

TABLE IX.

Tube.	Method I.				Method II.				Difference of differences $(a - b)$.
	Observed resistance.	Theoretical resistance, international ohms.	Observed − theoretical. $(a.)$	Difference from mean.	Observed resistance.	Theoretical resistance, international ohms.	Observed − theoretical. $(b.)$	Difference from mean.	
M	$1\cdot00005_2$	$\cdot99994_6$	$\cdot00010_6$	$+2_1$	$1\cdot00116_4$	$1\cdot00107_4$	$\cdot00009_0$	$+1_6$	$+1_6$
P	$\cdot99938_4$	$\cdot99930_4$	$\cdot00008_0$	-0_5	$1\cdot00047_0$	$1\cdot00040_1$	$\cdot00006_9$	-1_5	$+1_1$
T	$\cdot99912_1$	$\cdot99904_6$	$\cdot00007_5$	-1_0	$1\cdot00027_8$	$1\cdot00018_7$	$\cdot00009_1$	$+0_7$	-1_6
U	$\cdot99912_9$	$\cdot99904_6$	$\cdot00008_3$	-0_2	$1\cdot00021_7$	$1\cdot00014_8$	$\cdot00006_9$	-1_5	$+1_4$
V	$1\cdot00043_3$	$1\cdot00036_1$	$\cdot00007_2$	-1_3	$1\cdot00146_2$	$1\cdot00137_5$	$\cdot00008_7$	$+0_3$	-1_5
W	$\cdot99915_2$	$\cdot99905_5$	$\cdot00009_7$	$+1_2$	$1\cdot00015_3$	$1\cdot00005_7$	$\cdot00009_6$	$+1_2$	$+0_1$
X	$1\cdot00006_3$	$\cdot99998_0$	$\cdot00008_3$	-0_2	$1\cdot00115_1$	$1\cdot00105_3$	$\cdot00009_8$	$+1_4$	-1_5
Y	$\cdot99926_3$	$\cdot99915_5$	$\cdot00010_8$	$+2_3$	$1\cdot00035_0$	$1\cdot00027_0$	$\cdot00008_0$	-0_4	$+2_8$
Z	$1\cdot00032_5$	$1\cdot00024_8$	$\cdot00007_7$	-0_8	$1\cdot00138_9$	$1\cdot00129_6$	$\cdot00009_3$	$+0_9$	-1_6
G	$1\cdot00034_2$	$1\cdot00027$	$\cdot00007$	-1	$1\cdot00113_5$	$1\cdot00108$	$\cdot00005$	-3	$+2$
S	$1\cdot00025_4$	$1\cdot00016$	$\cdot00009$	$+1$	$1\cdot00105_7$	$1\cdot00096$	$\cdot00010$	$+2$	-1
		Mean . .	$0\cdot00008_5$			Mean . .	$0\cdot00008_4$		
			Mean of means . . $0\cdot00008_5$						

Thus we may write

Resistance of 1 international ohm — resistance of unit assumed as = to 10^9 C.G.S. units = $0\cdot00008_5$.

Taking the mean value as representing the international ohm, the evaluation of the standards becomes :

Standard tube.	Theoretical resistance in international ohms.	Observed resistance in international ohms.
M	$\cdot99994_6$	$\cdot99996_7$
P	$\cdot99930_4$	$\cdot99929_9$
T	$\cdot99904_6$	$\cdot99903_7$
U	$\cdot99904_6$	$\cdot99904_4$
V	$1\cdot00036_1$	$1\cdot00034_8$
W	$\cdot99905_5$	$\cdot99906_7$
X	$\cdot99998_0$	$\cdot99997_8$
Y	$\cdot99915_5$	$\cdot99917_8$
Z	$1\cdot00024_8$	$1\cdot00024_0$
G	$1\cdot00027$	$1\cdot00025_7$
S	$1\cdot00016$	$1\cdot00016_9$

with a probable error, as evaluated from the differences, of $\pm$ 0·0008 per cent.

That the differences (*a*) — (*b*) do not agree is not surprising. Apart from the experimental errors, the constant $f = 0{\cdot}795$ is a mean value only. Its magnitude must certainly depend to some extent on the ratio of the outer diameter of the tube to that of the bore, in other words, the "flange effect." Measurement shows this ratio to be approximately equal for the tubes, but the exact measurements are without interest, since the law of variation is not known. Again, the sections are not truly circular, and f is dependent on the contour. The agreement between the differences (*a*) and (*b*) may therefore be regarded as sufficiently close. It will be observed that the resulting sums of the differences are practically identical, from which it follows that if the various values of f were evaluated from the difference of the resistances in Methods I. and II., the mean f would have been identical with 0·795 to 0·1 per cent. This satisfactory result is no doubt due, in a large measure, to the number of standards employed.

Method No. III.—Measurement of Resistance by the Carey Foster Bridge.

In order to preserve continuity with the work of previous observers, and also to better decide the most advantageous method of measuring the resistance, observations were made by the Carey Foster Bridge. The method adopted was very similar to that of Lord RAYLEIGH in 1882 and of Mr. GLAZEBROOK in 1888. It is also of interest to note that the standard coils and the bridge (that designed by Dr. FLEMING for the British Association) were the same as employed by those observers.

Different connections were now requisite. The standard tube was terminated at each end by a glass vessel with three apertures. These are shown in the figure (fig. 16). The bungs employed were treated as before, the necessary tests for insulation resistance being made before proceeding with an observation.

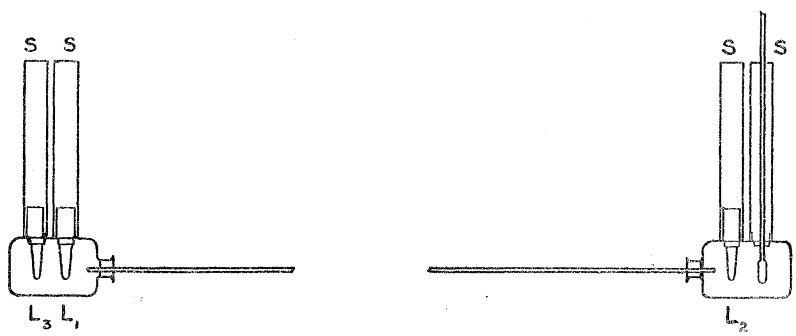

Fig. 16.

Three platinum thimbles $4\frac{1}{2}$ centims. long and $1\frac{1}{4}$ centims. in diameter at the orifice were inserted in ebonite collars and fixed in position at L_1, L_2, and L_3 as shown. Rubber sheaths passed from the necks of the vessels to the outside of the ice box. These sheaths were lined with glass tubes to give them greater rigidity, and also to ensure cleanliness of the interior. In the spare aperture a sensitive thermometer was placed, but the arrangements at the neck were such that a platinum thimble could be inserted if necessary. Shellac varnish was applied to all junctions to ensure good insulation of the mercury in the tube and vessels. Long copper leads were made so as to connect any two of the thimbles to two mercury cups of the Carey Foster bridge, one end of each lead being shaped so as to fit any one of the thimbles. Two compensating leads, also of copper, connected the two opposite mercury cups to the standard coil employed, the difference in resistance of the leads on opposite sides of the bridge amounting to about 0·00006 ohm.

Each platinum thimble had its entire surface amalgamated in the following manner. The platinum was first scoured with acids, and afterwards heated to redness in a Bunsen flame. Platinisation was then effected by the passage of a current of electricity through a solution of platinum chloride containing a small amount of lead acetate. Afterwards the platinum thimbles were heated to redness, and plunged whilst hot into mercury. Amalgamation immediately resulted, and thoroughly good contact between the copper and the mercury was thus secured. When the platinum thimbles were in position in the glass vessels, the distance from one end of the standard tube to the nearest platinum thimble was $1\frac{1}{2}$ centims.

The tube and vessels being filled with pure mercury, and the necessary accessories fitted, the whole was placed in the ice box, and the first observation of resistance made 12 hours afterwards. The ice was packed well about the sheaths, the upper surface being quite 8 centims. above the necks of the vessels. The copper leads were introduced into L_1 and L_2, cotton wool forming a collar about the rods at the point of emergence from the sheaths.

The necessary interval having elapsed, the temperature of the mercury in the vessel containing the thermometer was read, and the leads connected to the mercury cups of the bridge, the standard coil and its leads being already in position. An observation was now taken of the difference

Resistance of mercury standard plus leads — Resistance of standard coils plus leads.

Immediately this measurement was completed, the lead immersed in L_2 was rapidly removed to L_3, the leads of the coil being at the same time disposed so as to eliminate the resistance of the coil from the measurements. The difference

Resistance of leads of mercury standard ∸ Resistance of leads of standard coil

was then observed on the bridge. In this way, assuming all of the platinum thimbles to be similar, the resistance of the leads and thimbles was eliminated. A reversal to the former conditions was now effected, and the previous observation confirmed.

The recorded temperature of the mercury in the end vessels was always considerably above 0° C.; it averaged 1·3° C. The temperature of part of the mercury in the tube was therefore higher than the ice-point. The results of the observations are as follows, the temperature recorded being that of the mercury in contact with the thermometer bulb.

TABLE X.—Erection of Standards by Method III.

Resistance Measurements by Carey Foster Bridge.

Mercury standard.	Temperature of mercury in end vessels.	(a) Method of erection III. Resistance of standard.	(b) Method of Erection II. Resistance of standard.	Difference. (a)−(b).
	° C.	ohms.	ohms.	ohm.
M	1·2	1·00120	1·00116	0·00004
P	1·1	1·00051	1·00047	0·00004
P	1·8	1·00055	1·00047	0·00008
T	1·4	1·00033	1·00028	0·00005
U	1·4	1·00026	1·00022	0·00004
V	1·4	1·00150	1·00146	0·00004
V	1·2	1·00149	1·00146	0·00003
W	1·2	1·00019	1·00015	0·00004
X	1·3	1·00121	1·00115	0·00006
Y	1·2	1·00040	1·00035	0·00005
Z	1·2	1·00143	1·00139	0·00004
G	1·4	1·00119	1·00114	0·00005
S	1·5	1·00111	1·00106	0·00005

For reference, the values of the resistances when the standards were erected by Method II. are given in column 4 of the above table; differences are tabulated in column 5. If the end corrections and temperatures were identical in the two cases, there should, of course, be no difference between the values (*a*) and (*b*). The two measurements of P, when the conditions of temperature were different, indicate that the observed differences are largely due to a temperature effect. Indeed, the conduction of heat through the leads (they were 0·6 centim. in diameter) forbids the attainment of a steady known temperature. When the leads were removed from the platinum thimbles, and the sheaths plugged with cotton wool, the mercury was found to fall in temperature to the ice-point. If, when the mercury was at this temperature, the leads were quickly reintroduced, and a measurement made, a slightly lower value for the standard resulted. Such an observation was, however, very unsatisfactory, and, in order to obtain more definite information, the following experiments were made.

The tube V was chosen for the observations. It was erected by Method III. as before, and left in the ice box for 12 hours previous to any measurement. No copper leads were, however, introduced, the sheaths being plugged with cotton wool. The thermometer, also, was removed, a platinum thimble being substituted for it. Into the four thimbles, thin platinum wires passed; the outer two of these were intended for battery leads, the inner two as potential points. The resistance of the standard, with these connections, was measured by the potentiometer. Its value was $1 \cdot 00146_2$ ohms.

The potential leads were now removed, and the thick copper leads used for the bridge introduced. These now served as potential leads. After an interval of 2 hours an observation was made. The value obtained by the potentiometer was $1 \cdot 00150_0$ ohms.

A quick change over to the Carey Foster bridge, and the value there was also determined as 1·00150 ohms. After an interval of 12 hours the resistance was again measured by the potentiometer. The value was $1 \cdot 00150_5$ ohms. The copper leads were then removed and the platinum wires re-introduced. After 2 hours a measurement of $1 \cdot 00146_5$ ohms resulted. The conclusion is obvious. All bridge measurements of mercury standards in which the copper leads have been thick, and, in consequence, the temperature of the mercury uncertain, require correcting. The magnitude of this correction depends on the particular conditions attaining during the measurements. This is also indicated by Mr. GLAZEBROOK, 'Phil. Trans.,' 1888, p. 375. No great importance is, in consequence, attached to the measurements made by the Carey Foster bridge. It is interesting to note, however, that the conduction of heat through leads is easily observable with other resistances than mercury standards. A resistance coil, if cooled below the temperature of the room and then introduced to a bridge for evaluation, must be observed immediately. After an interval of a few minutes a different value may be obtained, owing to the conduction of heat through the leads to the coil. With a difference of temperature of 8° C. and employing a coil of platinum silver imbedded in paraffin wax, a difference in the values of 0·01 per cent. has often been obtained, the temperature of the liquid, in which the coil is immersed, remaining constant during the observations. With coils made of copper, no readable difference of temperature between the coil and the external contacts of the leads is admissible. In such cases, the whole must be immersed in an oil-bath. The influence of this conduction effect is considerable when determinations of temperature coefficients are made, and has probably led to many inaccurate determinations.

Purification of Mercury.

The whole of the mercury employed had been subjected to the following treatment. It was taken from the iron bottle in which it was delivered, squeezed through wash-leather, and passed in a finely divided condition through dilute nitric acid, and afterwards through distilled water. The acid and water were contained in glass tubes about 6 feet long. The mercury was then distilled in vacuo, and finally re-distilled in vacuo in an apparatus free from rubber in any form, so that glass alone came into contact with the mercury. In some of the determinations, when the standards were erected by Method II., the mercury employed had been but once distilled. No difference in the results was, however, found.

A Determination of the Temperature Coefficient of the Resistance of—

1. *Mercury in Jena 16''' glass.*
2. *Mercury in Verre dur glass.*
3. *A constant volume of mercury.*

Two very excellent series of observations on the Temperature Coefficient of Mercury have been made in the past by GUILLAUME,* and by KREICHGAUER and JÄGER.† On p. 112 it is shown that no very great difference exists between the two results. Nevertheless, the arrangements already dealt with for the resistance measurements of the standards were so convenient for such a determination that a series of observations (the temperature ranging from 0° C. to 22° C.) were made.

Owing to its high temperature coefficient, the resistance of a mercury column increases approximately 1 per cent. for an increment of temperature of 10° C., so that in conducting the measurements some difficulty arises if it be desired to make all the comparisons with one standard coil. The Kelvin bridge is, however, particularly suited for such measurements. In the case previously dealt with, the ratio arms R and S (see fig. 8) consist of 1000-ohm coils. The shunt necessary to alter the value of QR/S by 2 per cent. is, therefore, not small, and need only be approximately known. (This assumes that the shunted coil is either R or S.) Also the variation of resistance of the copper leads, owing to varying temperature, is known, since the resistance of the leads is measured immediately after that of the standard. At any temperature, therefore, the same degree of accuracy as before can be attained, even though the extreme measurements vary by 3 or 4 per cent.

The maintenance of the temperature of the standard so as to remain constant to 0°·01 C. for an hour or more, is an apparent difficulty only. Formerly, the tube was immersed in ice, and an interval of 5 hours was considered necessary before a measurement could be recorded. The conditions now, however, are changed. Liquid contact is secured, and this, assisted by violent agitation of the liquid, promotes uniformity of temperature. The interval of constancy therefore essential in order that the temperature of the mercury may be regarded as identical with that of the bath is much smaller than before. If the temperature can be regulated so as not to vary by more than 0°·005 C. in an hour, and also, if this temperature can be accurately read, then the conditions may be regarded as ideal. Such ideal conditions were practically realised, as will afterwards be shown.

The most accurate observations of a number at different temperatures will certainly be those at the ice-point, since not only is the temperature steady, and absolutely known, but, in addition, the observations can be made with more leisure. The mercury columns chosen were therefore such that at the high temperatures

* M. C. E. GUILLAUME, 'Bureau International des Poids et Mesures,' 1889–1890.

† 'WIEDEMANN'S Annalen,' vol. 47, 1892.

they had a nominal value of 1 ohm, while at 0° C. their values were considerably different to the combination value of the bridge coils.

Each of the tubes U and M was somewhat scratched near one end. Fearing that these scratch marks might develop into fractures, a length of about 1 centim. was broken off each tube and the new ends ground as before. The resistance of these two tubes was thus made equal to 1 ohm at about 30° C., their values at 0° C. being approximately 0·97 ohm.

The two tubes were erected as per Method II. already described, and during the cycle of temperature no change resulted in the purity of the mercury. Evidence of this statement is furnished by observations taken during the cycle; also, the initial resistance at the ice-point was identical with that observed when the cycle of temperature had been completed.

Originally it was intended to immerse the tubes in petroleum, but had this been done, final observations at 0° C. would have been impossible. Water was therefore used throughout, the insulation resistance being carefully measured at each temperature. The lowest value obtained for this was a little greater than 3 megohms.

The capacity of the bath was 8 gallons, and the lagging of cork dust was retained. The liquid was well stirred by means of a vane secured at one end of the bath, and in communication with the opposite end by means of a long brass tube. A small electric motor supplied the power. The efficiency of the stirring was tested by reading the temperature at different parts of the bath. The result was very satisfactory.

To secure different steady temperatures, several automatic devices seemed possible. The constancy desired (maximum variation of 0°·005 C.) was, however, beyond the limits of any of them. Eventually, capital results were obtained by allowing cold (or warm, according to the desired temperature) drops of water to fall into the bath. The water was contained in a funnel supported above the box, and the stream could be regulated by means of a tap. The approximate temperature was indicated by an open-scale thermometer supported vertically. This was read with a cathetometer telescope, a change of temperature of 0°·005 C. being easily discerned. With a little practice, temperatures between 0° C. and 24° C. could be maintained for several hours without the maximum variation exceeding this amount. Indeed, for the second set of observations, a temperature approximating to 5° C. was maintained for 4 hours, the greatest variation in temperature being that indicated. The mercury within the tube responded to changes of temperature very rapidly, however, and a few observations showed that it was only necessary to maintain a steady temperature for half an hour or even less.

It was desirable that the thermometer employed should be capable of being read to 0°·01 C. with ease and without the employment of a telescope. One of the thermometers presented to the Laboratory by the widow of the late Mr. Sworn, and numbered 2221, answered these requirements. The stem of this thermometer is divided into centimetres and millimetres, 1 centim. being equivalent to about 0°·6 C. Both before and after the observations, the thermometer was calibrated and tested under the direction of Dr. J. A. HARKER, the greatest difference between the mean and the observed values being 0°·01 C. All recorded temperatures are on the hydrogen scale, the resistance being expressed in the same units as before. In the tables, those measurements marked with an asterisk were taken when descending the scale of temperature.

The results are as follows :—

TABLE XI.—Mercury Standard U, Jena Glass 16'''.

Temperature. Hydrogen scale.	Resistance.	Temperature. Hydrogen scale.	Resistance.
° C.	ohm.	° C.	ohm.
0 ·0	0 ·97357$_7$	13 ·29	0 ·98514$_7$
0 ·0	0 ·97357$_8$	13 ·34	0 ·98518$_6$
0 ·0	0 ·97358$_3$	13 ·15	0 ·98502$_5$
0 ·0*	0 ·97358$_0$	13 ·21	0 ·98507$_9$
		13 ·33	0 ·98518$_2$
5 ·23	0 ·97807$_8$	13 ·14	0 ·98501$_9$
5 ·31	0 ·97815$_4$	13 ·22	0 ·98508$_6$
6 ·06	0 ·97880$_9$		
6 ·15	0 ·97889$_3$	17 ·71	0 ·98907$_3$
6 ·12	0 ·97886$_1$	17 ·71	0 ·98907$_3$
		17 ·71	0 ·98907$_7$
10 ·07	0 ·98232$_1$	17 ·71	0 ·98907$_9$
10 ·10	0 ·98235$_2$	17 ·71	0 ·98907$_9$
10 ·08	0 ·98233$_7$	17 ·99*	0 ·98932$_8$
10 ·09	0 ·98234$_3$	17 ·99*	0 ·98933$_0$
10 ·11	0 ·98235$_5$		
9 ·79*	0 ·98206$_7$	24 ·04	0 ·99476$_3$
9 ·80*	0 ·98206$_0$	24 ·01	0 ·99474$_6$
		24 ·04	0 ·99477$_7$
		24 ·14	0 ·99488$_0$
		24 ·12	0 ·99485$_3$

It should be noted that observations at approximately the same temperature represent, in general, an interval of several hours. Thus at 17·71° C. five observations are recorded, but 5 hours elapsed between the first and last of these. The results give thirty-five equations connecting resistance and temperature. Assuming the resistance to be such a function of the temperature that it is expressible in the form $R_T = R_0(1 + \alpha T + \beta T^2)$, the equations enable α and β to be found.

In evaluating these quantities, twice the weight has been given to the four observations at the ice-point. The three normal equations resulting by the method of least squares are

$$\text{(A)} \quad 39x + 436{\cdot}47a + 7207{\cdot}9363b = 0{\cdot}381818,$$
$$\text{(B)} \quad 436{\cdot}47x + 7207{\cdot}9363a + 133386{\cdot}322b = 6{\cdot}3182355,$$
$$\text{(C)} \quad 7207{\cdot}9363x + 133386{\cdot}322a + 2670818{\cdot}73b = 117122{\cdot}320,$$

where $x = R_0 - 0{\cdot}973576$, $a = 0{\cdot}973576\alpha$, and $b = 0{\cdot}973576\beta$, and finally

$a = 85693 \times 10^{-8}$, $b = 102998 \times 10^{-11}$, and $x = +94 \times 10^{-7}$. The resulting equation

$$R_T = R_0[1 + 0{\cdot}00088018T + 0{\cdot}00000105793T^2]$$

represents the change of resistance with temperature of mercury in Jena 16‴ glass. The differences "calculated values — observed values" are given in the accompanying table.

TABLE XII.—Mercury Standard U, Jena Glass 16‴.

Calculated — Observed Values.

$$U_T = U_0 + {\cdot}00085693T + {\cdot}00000102998T^2.$$

Temperature. Hydrogen scale.	Calculated value. $U_T - U_0$.	Observed value. $U_T - U_0$.	Residual errors calculated — observed. Parts in one million.
° C.			
0·0	·000000	— ·000001	+ ·000001
0·0	·000000	— ·000001	+ ·000001
0·0	·000000	— ·000002	+ ·000002
0·0	·000000	— ·000002	+ ·000002
0·0	·000000	+ ·000004	— ·000004
0·0	·000000	+ ·000004	— ·000004
0·0*	·000000	+ ·000001	— ·000001
0·0*	·000000	+ ·000001	— ·000001
5·23	·004510	+ ·004499	+ ·000011
5·31	·004579	+ ·004575	+ ·000004
6·06	·005231	+ ·005230	+ ·000001
6·15	·005309	+ ·005314	— ·000005
6·12	·005283	+ ·005282	+ ·000001
10·07	·008734	+ ·008742	— ·000008
10·10	·008759	+ ·008773	— ·000014
10·08	·008743	+ ·008758	— ·000015
10·09	·008751	+ ·008764	— ·000013
10·11	·008769	+ ·008776	— ·000007
9·79*	·008488	+ ·008481	+ ·000007
9·80*	·008497	+ ·008488	+ ·000009
13·29	·011571	+ ·011568	+ ·000003
13·34	·011615	+ ·011607	+ ·000008
13·15	·011447	+ ·011446	+ ·000001
13·21	·011500	+ ·011500	+ ·000000
13·33	·011606	+ ·011603	+ ·000003
13·14	·011438	+ ·011440	— ·000002
13·22	·011509	+ ·011507	+ ·000002
17·71	·015499	+ ·015494	+ ·000005
17·71	·015499	+ ·015494	+ ·000005
17·71	·015499	+ ·015508	+ ·000001
17·71	·015499	+ ·015500	— ·000001
17·71	·015499	+ ·015500	— ·000001
17·99*	·015750	+ ·015749	+ ·000001
17·99*	·015750	+ ·015751	— ·000001
24·04	·021196	+ ·021184	+ ·000012
24·01	·021169	+ ·021167	+ ·000002
24·04	·021196	+ ·021198	— ·000002
24·14	·021287	+ ·021301	— ·000014
24·12	·021268	+ ·021274	— ·000006
		Total . .	— ·000099 + ·000082 + ·000016†
		Sum . . .	— ·000001

* These observations were taken after the standard had been raised to the temperature 24° C.

† x has been taken equal to + ·000009 instead of + $\cdot000009_4$ in above calculations. Hence the differences are all too small by 39 × ·0000004 = ·000016.

The sum of the squares of the residual errors is 15·71, and since twice the weight was given to the observations at 0° C., the number of observations may be regarded as 39. Hence the probable error of any single determination is $0·6745\ (15·71/38)^{\frac{1}{2}}$, which is equal to $\pm\ 0·43$, a little more than four parts in one million. If the residual errors be written down in the order of their magnitude, the central error is three parts in 1,000,000. With the aid of the data in the three normal equations, the probable errors of the coefficients α and β may now be calculated as follows:—For

$$\alpha,\ \pm 7 \times 10^{-7}\,(39 - 436·5^2/7207·9)^{\frac{1}{2}} = \pm\, 2 \times 10^{-7},$$

and for

$$\beta,\ \pm 7 \times 10^{-7}\,(2323·2)^{\frac{1}{2}} = \pm\, 145 \times 10^{-10}.$$

The coefficient of cubical expansion of Jena 16‴ glass is 231×10^{-7}. Assuming the glass to be isotropic, the variation of resistance with temperature of a constant volume of mercury as deduced from the previous equation, is

$$R_T = R_0\,[1 + 0·00088788T + 0·0000010564T^2],$$

the probable errors being as before.

Standard M. Verre dur glass. Similar remarks apply to the observations with this tube. Table XIII. gives the results of the resistance measurements.

TABLE XIII.—Mercury Standard M. Verre dur Glass.

Temperature. Hydrogen scale.	Resistance.	Temperature. Hydrogen scale.	Resistance.
° C.	ohm.	° C.	ohm.
0 ·0	$\cdot97178_3$	13 ·28	$\cdot98332_9$
0 ·0	$\cdot97178_7$	13 ·32	$\cdot98335_8$
0 ·0	$\cdot97178_9$	13 ·29	$\cdot98334_4$
0 ·0*	$\cdot97179_1$	13 ·29	$\cdot98333_6$
		13 ·31	$\cdot98335_2$
5 ·12	$\cdot97619_1$	13 ·31	$\cdot98334_7$
5 ·09	$\cdot97617_1$		
5 ·17	$\cdot97624_2$	17 ·73	$\cdot98727_2$
5 ·23	$\cdot97629_1$	17 ·73	$\cdot98727_1$
5 ·21	$\cdot97627_4$	17 ·73	$\cdot98727_3$
6 ·16	$\cdot97709_5$	17 ·72	$\cdot98725_5$
		17 ·70	$\cdot98724_1$
9 ·97	$\cdot98041_8$	17 ·99*	$\cdot98749_8$
10 ·01	$\cdot98045_6$	17 ·99*	$\cdot98750_2$
10 ·02	$\cdot98046_3$		
10 ·03	$\cdot98047_0$	24 ·36	$\cdot99322_4$
9 ·79*	$\cdot98026_1$	24 ·40	$\cdot99326_0$
9 ·83*	$\cdot98029_7$	24 ·33	$\cdot99319_5$
9 ·81*	$\cdot98028_3$	24 ·25	$\cdot99312_6$
		24 ·20	$\cdot99308_7$
		24 ·15	$\cdot99303_4$

As before, twice the weight has been given to the observations at the ice-point. The three normal equations obtained by the method of least squares are

$$\begin{aligned}
&(A)\quad 40x + 451·52a + 7677·2174b = ·393913,\\
&(B)\quad 451·52x + 7677·2174a + 147255·962b = 6·7149215.\\
&(C)\quad 7677·2174x + 147255·962a + 3049262·28b = 129·0255119,
\end{aligned}$$

where $x = R_0 - \cdot 971796$, $a = 0\cdot 971796\alpha$, and $b = 0\cdot 971796\beta$, and finally $a = 85553 \times 10^{-8}$, $b = 100286 \times 10^{-11}$, and $x = -1875 \times 10^{-9}$. Whence

$$R_T = R_0\,[1 + 0\cdot 00088036T + 0\cdot 00000103094T^2].$$

This equation represents the change of resistance with temperature of a mercury standard, the column of mercury being contained in a Verre dur glass tube, the limits of temperature being 0° C. and 24° C.

TABLE XIV.—Standard M. Verre dur Glass.

Calculated — Observed values.

$$M_T = M_0 + \cdot 00085553_1 T + \cdot 00000100286T^2.$$

Temperature. Hydrogen scale.	Calculated value. $M_T - M_0$.	Observed values. $M_T - M_0$.	Residual errors calculated — observed. Parts in one million.
° C.			
0 ·0	0 ·000000	−0 ·000005	+0 ·000005
0 ·0	0 ·000000	−0 ·000005	+0 ·000005
0 ·0	0 ·000000	−0 ·000001	+0 ·000001
0 ·0	0 ·000000	−0 ·000001	+0 ·000001
0 ·0	0 ·000000	+0 ·000001	−0 ·000001
0 ·0	0 ·000000	+0 ·000001	−0 ·000001
0 ·0*	0 ·000000	+0 ·000003	−0 ·000003
0 ·0*	0 ·000000	+0 ·000003	−0 ·000003
5 ·12	0 ·004407	+0 ·004403	+0 ·000004
5 ·09	0 ·004381	+0 ·004383	−0 ·000002
5 ·17	0 ·004450	+0 ·004454	−0 ·000004
5 ·23	0 ·004502	+0 ·004502	+0 ·000000
5 ·21	0 ·004484	+0 ·004484	+0 ·000000
6 ·16	0 ·005308	+0 ·005307	+0 ·000001
9 ·97	0 ·008629	+0 ·008630	−0 ·000001
10 ·01	0 ·008664	+0 ·008668	−0 ·000004
10 ·02	0 ·008673	+0 ·008675	−0 ·000002
10 ·03	0 ·008682	+0 ·008682	+0 ·000000
9 ·79*	0 ·008472	+0 ·008473	−0 ·000001
9 ·83*	0 ·008507	+0 ·008509	−0 ·000002
9 ·81*	0 ·008489	+0 ·008495	−0 ·000006
13 ·28	0 ·011538	+0 ·011541	−0 ·000003
13 ·22	0 ·011574	+0 ·011570	+0 ·000004
13 ·29	0 ·011547	+0 ·011556	−0 ·000009
13 ·29	0 ·011547	+0 ·011548	−0 ·000001
13 ·31	0 ·011565	+0 ·011564	+0 ·000001
13 ·31	0 ·011565	+0 ·011559	+0 ·000006
17 ·73	0 ·015484	+0 ·015484	+0 ·000000
17 ·73	0 ·015484	+0 ·015483	+0 ·000001
17 ·73	0 ·015484	+0 ·015485	−0 ·000001
17 ·72	0 ·015475	+0 ·015467	+0 ·000008
17 ·70	0 ·015457	+0 ·015453	+0 ·000004
17 ·99*	0 ·015716	+0 ·015710	+0 ·000006
17 ·99*	0 ·015716	+0 ·015714	+0 ·000002
24 ·36	0 ·021436	+0 ·021436	+0 ·000000
24 ·40	0 ·021472	+0 ·021472	+0 ·000000
24 ·33	0 ·021409	+0 ·021407	+0 ·000002
24 ·25	0 ·021336	+0 ·021338	−0 ·000002
24 ·20	0 ·021291	+0 ·021299	−0 ·000008
24 ·15	0 ·021246	+0 ·021246	+0 ·000000
		Total . .	−0 ·000054 +0 ·000051 +0 ·000005†
		Sum . . .	+0 ·000002

* Observations taken after the standard had been raised to the temperature 24° C.

† Due to x being taken as $-\cdot 000002$.

The sum of the squares of the residual errors is 5·9. Hence the probable error of any single observation is $0·6745\,(5·09/39)^{\frac{1}{2}} = \pm\,0·243$, a little more than two parts in a million. If the residual errors be written down in the order of their magnitude, the central error is also two parts in a million. The calculated probable error of α is $\pm\,1 \times 10^{-7}$ and of $\beta \pm 8 \times 10^{-9}$.

The coefficient of cubical expansion of Verre dur glass being 222×10^{-7}, the equation connecting resistance and temperature of a constant volume of mercury is

$$R_T = R_0\,[1 + 0·00088776T + 0·0000010376T^2],$$

the glass being assumed as isotropic. The equation derived from measurements when the standard U was employed is

$$R_T = R_0\,[1 + 0·00088788T + 0·0000010564T^2].$$

The greatest difference between the values derived from the two equations is 0·0003 per cent. at 10° C. and 0·0010 per cent. at 20° C.

For a constant volume of mercury the equations obtained by M. GUILLAUME were*

$$(a) \quad R_T = R_0\,[1 + 0·00088745T + 0·0000010181T^2],$$
$$(b) \quad R_T = R_0\,[1 + 0·00088879T + 0·0000010022T^2],$$

the temperature being expressed on the hydrogen scale.

The equation obtained by KREICHGAUER and JÄGER† is

$$R_T = R_0\,[1 + 0·0008827T + 0·00000126T^2].$$

Table XV. gives the values, as calculated from these five equations, of the resistance of a constant volume of mercury at 10° C. and 20° C., the resistance at 0° C. being 1 ohm.

TABLE XV.

Equation.	Value of resistance at 10° C.	Value of resistance at 20° C.
GUILLAUME (a)	1·008976	1·018156
„ (b)	1·008988	1·018177
KREICHGAUER and JÄGER	1·008953	1·018158
National Physical Laboratory, M	1·008981	1·018170
„ „ „ U	1·008984	1·018180

Probably the introduction of γT^3 in the equations would bring the results even nearer together. A difference of 0·001 per cent. corresponds to a difference in temperature of 0·01° C.

* M. C. E. GUILLAUME, 'Bureau International des Poids et Mesures,' 1892.

† 'WIEDEMANN'S Annalen,' vol. 47, 1892.

Summary.

It is not difficult to summarise the results. The difficulty of defining and constructing a practical invariable mercury standard of resistance has been shown to be rather under- than over-estimated. The possibility that the axis of the tube may be undulating seems difficult to remove, though the final results indicate that its effect is small. The presence of films of moisture and of air is disconcerting, though again the results indicate the constancy of these under definite conditions. Of the methods chosen to measure the resistance of mercury standards, that of the Kelvin double bridge is certainly best, the tube being erected as indicated in Method II. As already mentioned for permanent work, all rubber may be removed, the tubes themselves being ground as stoppers. The temperature of observations is advisably that of melting ice, for while there is little uncertainty concerning the equality of temperature of the mercury and a well-stirred liquid, the thermometry then plays a considerable part in the determinations. The change in resistance is determinable to 0·0001 per cent., equivalent to one-thousandth of a degree on the centigrade scale.

The primary objects in view have, it would appear, been achieved with a fair measure of success. The eleven mercury standards constructed from theoretical considerations enable a resistance to be evaluated in international ohms with an accuracy of at least 0·001 per cent. This is evident from Table IX. The international ohms, constructed at the Phys.-Tech. Reichsanstalt, have indirectly been compared with those constructed at the National Physical Laboratory. In the Report of the British Association (Section A) for 1903, the ratio of the unit of resistance employed at the Phys.-Tech. Reichsanstalt to that of the National Physical Laboratory is discussed. A table of measurements of certain manganin coils, ranging from 0·1 ohm to 10,000 ohms, is given. These measurements were made partly at the Reichsanstalt (the corresponding values being international ohms), and in part at Teddington (the values, presumably, being expressed in terms of the absolute ohm [10^9 C.G.S. units]). For the sake of completeness, the table is given here :—

TABLE XVI.—Results of Measurements of Various Coils at the Reichsanstalt and at the National Physical Laboratory, March, 1903.

Coil No.	National Physical Laboratory value at 17° C.	Value deduced from Reichsanstalt certificate at 17° C.	National Physical Laboratory value — Reichsanstalt value. Percentage difference
			per cent.
2352	·100007	·099996	·011
2351	1 ·00011	1 ·00001	·010
780	1 ·00001	·99991	·010
738	9 ·9994$_5$	9 ·9985	·009$_5$
2450	100 ·004	99 ·993	·011
2449	1000 ·06	999 ·96	·010
2448	10000 ·9	9999 ·8	·011
		Mean . .	·010$_4$

It is evident from these observations that a difference of $\cdot 010_4$ per cent. exists; or, to express the difference fully,

Resistance of Reichsanstalt unit — Resistance of N.P.L. unit = $\cdot 00010_4$ ohm.

The eleven mercury standards constructed show that

Resistance of international ohm — Resistance of N.P.L. unit = $\cdot 00008_5$ ohm.

Hence

$$\left.\begin{array}{l}\text{Resistance of unit employed at the}\\ \quad\text{Reichsanstalt as 1 international}\\ \quad\text{ohm}\end{array}\right\} - \left\{\begin{array}{l}\text{Resistance of unit employed at the}\\ \quad\text{National Physical Laboratory as}\\ \quad\text{1 international ohm}\end{array}\right.$$

$$= \cdot 00001_9 \text{ ohm.}$$

So that an approximate difference of two parts in one hundred thousand exists between the units derived at the two laboratories.

Assuming that all the changes in the B.A. coils have been rightly interpreted, and that the correct ratio of the B.A. unit to the absolute ohm was fixed upon (see p. 115), then for N.P.L. unit absolute ohm may be written. Hence, from the observations at Teddington,

Resistance of international unit — Resistance of absolute unit = $\cdot 000085$ ohm.

In other words, the absolute ohm is represented by a column of mercury 106·292 centims. long, of 14·4510 grammes mass, at a temperature of 0°·0 C.

The ratio of the B.A. unit to the absolute ohm was, however, fixed to ·01 per cent. only, so that the above figures merely indicate the order of the difference. The relation of the two units is, however, of interest; more especially so, since it will shortly be possible to again determine the coils in absolute measure.

A comparison with the results obtained by Dr. GLAZEBROOK ('Phil. Trans.,' A, 1888) for the specific resistance of mercury is also possible.

The specific resistance was determined in terms of the B.A. unit, the coils employed being those known as Flat, F, G, and H, and belonging to the Association. These four coils are now in the possession of the National Physical Laboratory, and although considerable changes in the values of the coils have manifested themselves since 1888, there is much evidence for the belief that the variations have been successfully interpreted ('B.A. Report,' Southport, 1903).

If it be assumed that the ratio of the B.A. unit to the absolute ohm is known, these coils enable absolute measurements of resistance to be made. On such an assumption, the resistances of the mercury standards dealt with in this paper have been determined. Similarly (but assuming a different ratio) Dr. GLAZEBROOK estimated the specific resistance of mercury in absolute measure. If the changes in value of the coils have been correctly noted, then the two results should be in perfect agreement within the limits of the errors of observation. Certain corrections have, of course, to be made.

The final result of Dr. GLAZEBROOK's investigation was that a column of mercury, 1 metre long and 1 sq. millim. in cross-section, had a resistance at 0° C. equal to 0·95352 B.A. unit; or, to put it differently, the length of a column of mercury of the same cross-section as before, and at the same temperature, having a resistance of 1 B.A. unit, was $104{\cdot}87_5$ centims. The number 0·98667 was adopted as the ratio of the B.A. unit to the absolute ohm. This number is the mean of several determinations made by Lord RAYLEIGH and Dr. GLAZEBROOK at the Cavendish Laboratory, Cambridge. The length of the mercury column representing the ohm was thus calculated to be $106{\cdot}29_1$ centims.

In order to compare the above result with that contained in this paper, several corrections have been introduced.

Correction (1).—The number adopted at the present time as representing the ratio of the B.A. unit to the absolute ohm is not that employed by Dr. GLAZEBROOK in 1888. The history of the change is contained in the reports of the British Association for 1890, 1891, and 1892. The British Association Committee in 1890 discussed the results of a number of investigations dealing with the subject. The decision finally arrived at was, that for practical purposes the resistance of a column of mercury, 106·3 centims. long, 1 sq. millim. in cross-section, and at a temperature of 0° C., was sufficiently near to the absolute ohm; also, that the number 0·9866 best expressed the ratio of the B.A. unit to the ohm. This number was adopted, and at the present time is still adhered to. In order to evaluate differences, the ratio has furthermore been assumed as equal to ·98660.

If this number correctly expresses the ratio, then the length of the mercury column having a resistance of 1 ohm, as calculated from Dr. GLAZEBROOK's observations, is $106{\cdot}29_8$ centims.

Correction (2).—The whole of the measurements in 1888 were made by the Carey Foster bridge, the mercury tubes being immersed in ice. The observations recorded on p. 105 of this report show that resistances so evaluated are liable to be greater than the true resistance at 0° C. Dr. GLAZEBROOK noted the temperatures of the end vessels when making comparisons; the average temperature was 1·3° C. A correction of − ·004 per cent. may therefore be safely applied. The length thus corrected is $106{\cdot}30_2$ centims.

Correction (3).—Shortly previous to the observations in 1888 some change took place in the standard coils already referred to. The coil "Flat" was found to have fallen ·01 per cent. in value relatively to the other coils. At the time the justified assumption was that Flat had changed, and that the others had remained constant. The value of Flat was therefore taken as ·01 per cent. lower than usual (p. 355, 'Phil. Trans.,' A, 1888). Intercomparisons in succeeding years rather negative this view of the change. The observations (a large number of coils were employed) show that the three coils assumed to be constant are liable to considerable fluctuations in value, while Flat appears to be a remarkably constant coil; this is shown in the 'B.A. Report' for 1903. Consequently, the values assigned to the resistances in 1888 were probably too small by 0·01 per cent. Correcting for this, the length of the mercury column becomes 106·29 centims. (the probable error of the investigation being ± 0·004 per cent., the third decimal figure has been discarded). The value given on p. 114 is 106·292 centims.; thus the two results are identical to 0·01 per cent.

The definitions of the mercury column representing the ohm are different in the two cases. In 1888 the definition necessitated a knowledge of the density of mercury, while at the present time it is in terms of a length and a mass. The value taken by Dr. GLAZEBROOK for the density of mercury at 0° C. was 13·5957 grammes per cubic centimetre; the difference in the two definitions is therefore small, being equal to one part in one hundred thousand. The identity of the results to 0·01 per cent. is not therefore affected.

Constancy of Mercury Standards.

Some information respecting the stability of the tubes may be gleaned from ice-point determinations of thermometers. The permanency of the mercury resistances depends entirely on the freedom of the glass from strain, or, if in a strained condition, the maintenance of that strain. Now the walls of a mercury tube are much thicker than the walls of a thermometer bulb. Probably, therefore, complete recovery from strain will be more difficult in the former case than the latter. On the other hand, the distorting forces to which a thermometer bulb is subject are much greater and more variable than those acting on a mercury standard. Under similar conditions, therefore, a mercury tube is probably not more liable to change than the bulb of a thermometer; also, the condition of a tube when a measurement of its resistance is being made is very similar to that of a thermometer which has been raised to the steam-point shortly before an observation of its zero. The constancy of such zero readings should therefore be comparable with the constancy of the resistance measurements. In the case of well-annealed Verre dur thermometers, Dr. J. A. HARKER, of the Thermometric Department of the Laboratory, assures the writer that the difference between such zero readings, spread over a considerable number of years, will not average, in the general case, more than a few hundredths of a degree centigrade. Since a change of 0·1° C. corresponds to an alteration in the capacity of the bulb of 0·002 per cent., changes of considerable magnitude in the standards of resistance are not anticipated.

The constancy of the mercury standards of the Phys.-Tech. Reichsanstalt is shown in vol. iv., 'Wissenschaftliche Abhandlungen der Phys.-Tech. Reichsanstalt.' The standards were constructed in 1890 and 1893, and no variation so great as 0·001 per cent. has up to the present (1903) been detected.

The whole of the observations connected with this investigation have been carried out at the National Physical Laboratory. As before stated, the work was first initiated by the Director in 1900, and the course followed is such as to preserve continuity with the past researches of the British Association. To the Director the author has to express his great indebtedness in all departments of the work. To his colleague, Mr. B. F. E. KEELING, the success of the linear measurements is entirely due, while the various members of the staff are thanked for their kindly interest and advice.

APPENDIX I.

Explanation of the Calibration Curves and Tables in Appendix.

All the calibration curves are plotted to approximately the same scale, the ordinate represented by the side of one square being equivalent to a change in cross-section of very nearly 0·5 per cent. The more correct scale is given in the following table. For each standard there is also given the distance from the etched mark to that end of the tube nearer to the mark. This distance is required both for the evaluation of μ' and for the calculation of the mean cross-sections of different portions of the tube. See Table III.

TABLE XVII.

Standard.	Length at 0° C.	Distance of mark from one end.	One scale division of ordinate of calibration curve represents a change in cross-section of—
	centims.	centims.	
M	59 ·0259	2 ·52	·50 per cent. from mean
P	63 ·4981	9 ·90	·48 ,, ,,
T	57 ·7249	4 ·82	·48 ,, ,,
U	62 ·0731	6 ·94	·50 ,, ,,
V	73 ·5000	3 ·48	·49 ,, ,,
W	75 ·9210	8 ·95	·50 ,, ,,
X	65 ·6338	11 ·38	·50 ,, ,,
Y	62 ·1867	11 ·98	·50 ,, ,,
Z	68 ·5199	13 ·08	·51 ,, ,,
G	116 ·507	17 ·40	·49 ,, ,,
S	119 ·472	18 ·58	·50 ,, ,,

APPENDIX II.

For each tube five values of μ' were calculated from five series of observations, the formula employed for each series being

$$\mu' = (\Sigma\lambda + a\lambda_a + b\lambda_b)(\Sigma 1/\lambda + a/\lambda_a + b/\lambda_b)/(n + a + b)^2,$$

in accordance with the text (see p. 59).

Throughout the whole of the observations on any one tube the mass of the mercury thread employed was constant. The observations were therefore combined and a sixth value of μ' calculated, the formula employed being

$$\mu' = (\Sigma\lambda + a_1\lambda_{a_1} + b_1\lambda_{b_1} + a_2\lambda_{a_2} + b_2\lambda_{b_2} + \ldots)(\Sigma 1/\lambda + a_1/\lambda_{a_1} + \ldots)/(n + a_1 + b_1 + a_2 + \ldots)^2.$$

Since the number of observations in this latter case is five times as great as those of each previous calculation, the probable error is less. This value of μ' has therefore been used in the various calculations (see column 4, Table XVIII.).

The calibration data for the standard W are given in full in Table XIX. Owing to the large amount of space similar tables for the remaining tubes would occupy, they are omitted.

In Table XIX. the lengths λ of the 5-centim. threads are recorded in the order of measurement. These lengths are the sum of those measured from the bases of the menisci, and the equivalent lengths of the menisci. The first column indicates the mean position of the thread. While it is unnecessary to evaluate λ to more than three decimal places, it may not be out of place to mention that the reciprocals must be taken to seven or eight places.

In Table XVIII. the mean value of μ'', the conical correction for the 5-centim. lengths, is given in column 5, and finally the product $\mu'\mu'' = \mu$. Thus the complete correction for conicality is obtained.

Calibration Curves.

In these, the extremities of the standard are indicated by a change in the thickness of the curve, while the position of the etched mark is indicated by a short vertical line lettered M. In each case the mean line of the standard portion of the curve is coincident with the line representing unit cross-section.

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TABLE XVIII.—Giving the Values of μ', μ'', and μ, for the various Standard Tubes. The Method of Evaluation is explained in the Introduction to the Appendix.

Mercury standard.	Series of calibration observations.	Calculated values of μ'.	Value of μ' obtained by combining the five sets of observations.	Mean value of μ''.	$\mu = \mu'\mu''$.
	No.				
M	1	1 ·000067			
	2	1 ·000066			
	3	1 ·000070	1 ·000067	1 ·000002	1 ·000069
	4	1 ·000068			
	5	1 ·000066			
P	1	1 ·000098			
	2	1 ·000100			
	3	1 ·000101	1 ·000099	1 ·000005	1 ·000104
	4	1 ·000101			
	5	1 ·000094			
T	1	1 ·000007			
	2	1 ·000006			
	3	1 ·000006	1 ·000007	1 ·000001	1 ·000008
	4	1 ·000007			
	5	1 ·000006			
U	1	1 ·000056			
	2	1 ·000055			
	3	1 ·000055	1 ·000055	1 ·000003	1 ·000058
	4	1 ·000055			
	5	1 ·000055			
V	1	1 ·000194			
	2	1 ·000193			
	3	1 ·000200	1 ·000197	1 ·000002	1 ·000199
	4	1 ·000197			
	5	1 ·000198			
W	1	1 ·000082			
	2	1 ·000078			
	3	1 ·000078	1 ·000081	1 ·000004	1 ·000085
	4	1 ·000083			
	5	1 ·000085			
X	1	1 ·000033			
	2	1 ·000031			
	3	1 ·000034	1 ·000032	1 ·000001	1 ·000033
	4	1 ·000033			
	5	1 ·000030			
Y	1	1 ·000125			
	2	1 ·000116			
	3	1 ·000117	1 ·000119	1 ·000004	1 ·000123
	4	1 ·000117			
	5	1 ·000118			
Z	1	1 ·000096			
	2	1 ·000097			
	3	1 ·000095	1 ·000095	1 ·000004	1 ·000099
	4	1 ·000094			
	5	1 ·000093			
G	1	1 ·000140			
	2	1 ·000141			
	3	1 ·000143	1 ·000140	1 ·000001	1 ·000141
	4	1 ·000138			
	5	1 ·000137			
S	1	1 ·000024			
	2	1 ·000020			
	3	1 ·000021	1 ·000022	1 ·000001	1 ·000023
	4	1 ·000022			
	5	1 ·000024			

TABLE XIX.—Mercury Standard W. Jena Glass 16'''.

The position of the mercury thread is stated relatively to a mark etched on the tube.

The tube has been cut at the distances − 8·95 and + 66·97 centims. from the mark.

Mean position of mercury thread.	Lengths of menisci. *l*.	Lengths of menisci. *r*.	Length of thread to bases of column.	L = thread length corrected for menisci.	$\frac{1}{L}$.
−20	·014	·016	5·025	5·040	·1984127
−15	·012	·013	4·979	4·992	·2003205
−10	·014	·015	4·964	4·979	·2008435
− 5	·016	·015	5·000	5·016	·1993620
0	·014	·016	4·927	4·942	·2023472
+ 5	·015	·012	4·885	4·899	·2041233
10	·017	·015	4·923	4·939	·2024701
15	·013	·014	4·926	4·940	·2024291
20	·012	·014	4·909	4·922	·2031694
25	·013	·015	4·882	4·896	·2042484
30	·014	·016	4·885	4·900	·2040816
35	·016	·016	4·927	4·943	·2023063
40	·015	·014	4·941	4·956	·2017756
45	·015	·017	4·906	4·922	·2031694
50	·015	·014	4·944	4·959	·2016535
55	·014	·016	4·994	5·009	·1996406
60	·016	·011	5·024	5·038	·1984915
65	·012	·015	5·010	5·024	·1990446
70	·014	·013	4·962	4·976	·2009646
−19	·014	·014	5·018	5·032	·1987281
−14	·013	·017	4·968	4·983	·2006823
− 9	·019	·011	4·973	4·988	·2004812
− 4	·015	·014	4·984	4·999	·2000400
+ 1	·013	·012	4·917	4·930	·2028398
6	·014	·017	4·884	4·900	·2040816
11	·011	·020	4·927	4·943	·2023063
16	·012	·018	4·918	4·933	·2027164
21	·016	·015	4·897	4·913	·2035416
26	·015	·013	4·876	4·890	·2044990
31	·013	·015	4·890	4·904	·2039152
36	·010	·017	4·939	4·953	·2018978
41	·015	·013	4·935	4·949	·2020610
46	·016	·014	4·908	4·923	·2031282
51	·017	·016	4·952	4·969	·2012477
56	·013	·016	5·003	5·018	·1992826
61	·014	·016	5·022	5·037	·1985309
66	·019	·013	4·994	5·010	·1996008
−18	·017	·012	5·009	5·024	·1990446
−13	·014	·015	4·957	4·972	·2011263
− 8	·020	·012	4·987	5·003	·1998801
− 3	·017	·014	4·970	4·986	·2005616
+ 2	·015	·013	4·904	4·918	·2033347
7	·014	·012	4·892	4·905	·2038736
12	·013	·018	4·930	4·946	·2021836
17	·015	·017	4·918	4·934	·2026753
22	·016	·017	4·894	4·911	·2036245
27	·018	·015	4·874	4·891	·2044572
32	·011	·015	4·900	4·913	·2035416
37	·017	·013	4·946	4·961	·2015723
42	·014	·014	4·926	4·940	·2024291
47	·016	·013	4·913	4·928	·2029221
52	·016	·016	4·965	4·981	·2007629
57	·013	·017	5·009	5·024	·1990446
62	·012	·014	5·028	5·041	·1983733
67	·014	·018	4·978	4·994	·2002403
−17	·010	·017	4·998	5·012	·1995211
−12	·012	·013	4·955	4·968	·2012882
− 7	·011	·019	5·001	5·016	·1993620
− 2	·015	·018	4·952	4·969	·2012477
+ 3	·015	·012	4·895	4·909	·2037075
8	·014	·015	4·900	4·915	·2034588
13	·014	·016	4·926	4·941	·2023882
18	·014	·016	4·911	4·926	·2030045
23	·016	·014	4·888	4·903	·2039568
28	·019	·013	4·873	4·889	·2045408
33	·014	·018	4·903	4·919	·2032934
38	·010	·018	4·951	4·965	·2014099
43	·017	·015	4·912	4·928	·2029221
48	·016	·012	4·923	4·937	·2025522
53	·013	·020	4·972	4·989	·2004410
58	·012	·017	5·012	5·027	·1989258
63	·018	·014	5·022	5·038	·1984915
68	·020	·010	4·965	4·980	·2008032
−16	·014	·012	4·988	5·001	·1999600
−11	·013	·015	4·954	4·968	·2012882
− 6	·013	·012	5·004	5·017	·1993223
− 1	·015	·011	4·939	4·952	·2019386
+ 4	·016	·014	4·885	4·900	·2040816
9	·011	·017	4·906	4·920	·2032520
14	·014	·017	4·921	4·937	·2025522
19	·013	·015	4·911	4·925	·2030457
24	·012	·015	4·884	4·898	·2041650
29	·017	·015	4·878	4·894	·2043318
34	·016	·013	4·918	4·933	·2027164
39	·014	·019	4·946	4·963	·2014910
44	·012	·016	4·910	4·924	·2030869
49	·011	·015	4·936	4·949	·2020610
54	·016	·014	4·984	4·999	·2000400
59	·017	·014	5·017	5·033	·1986887
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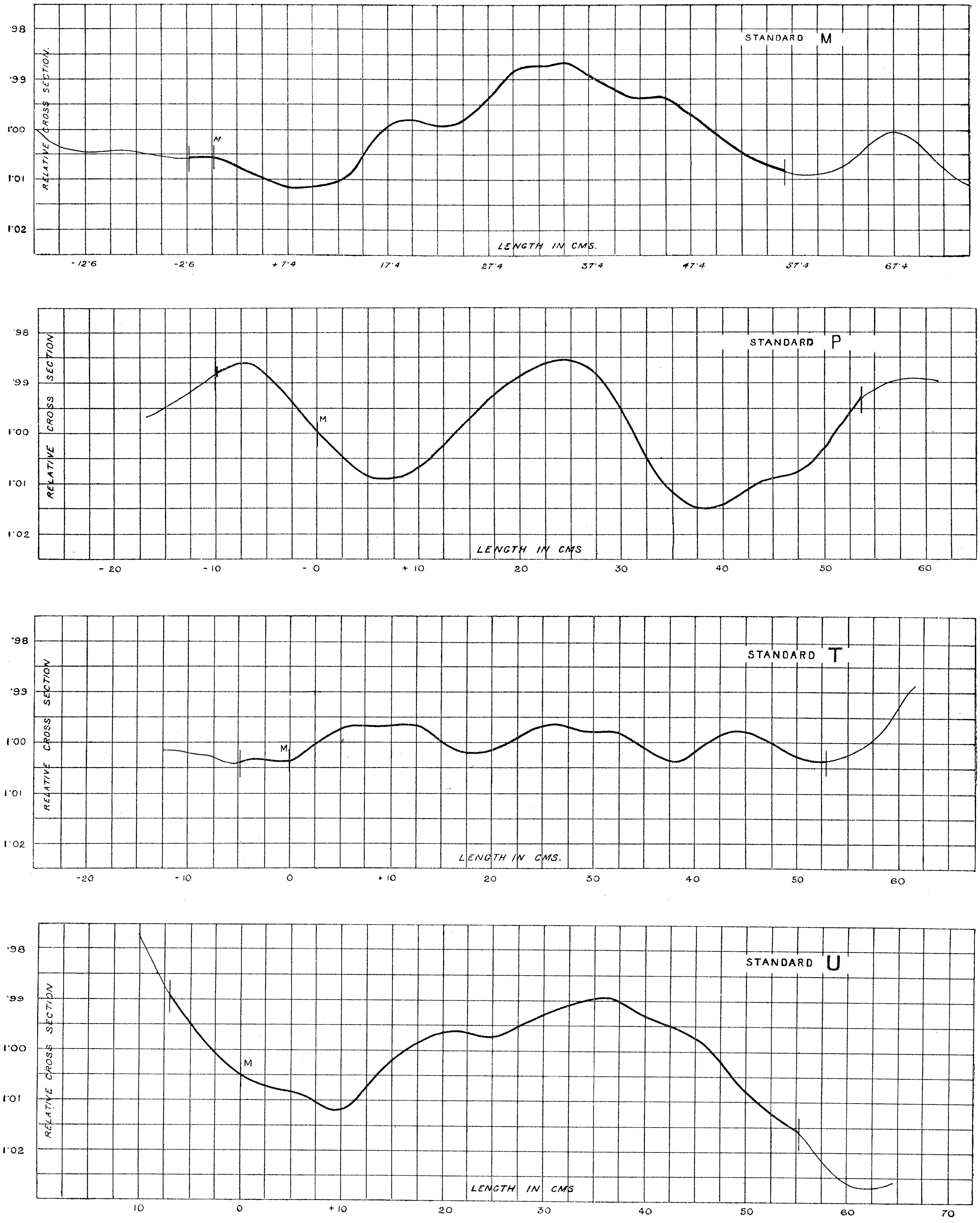

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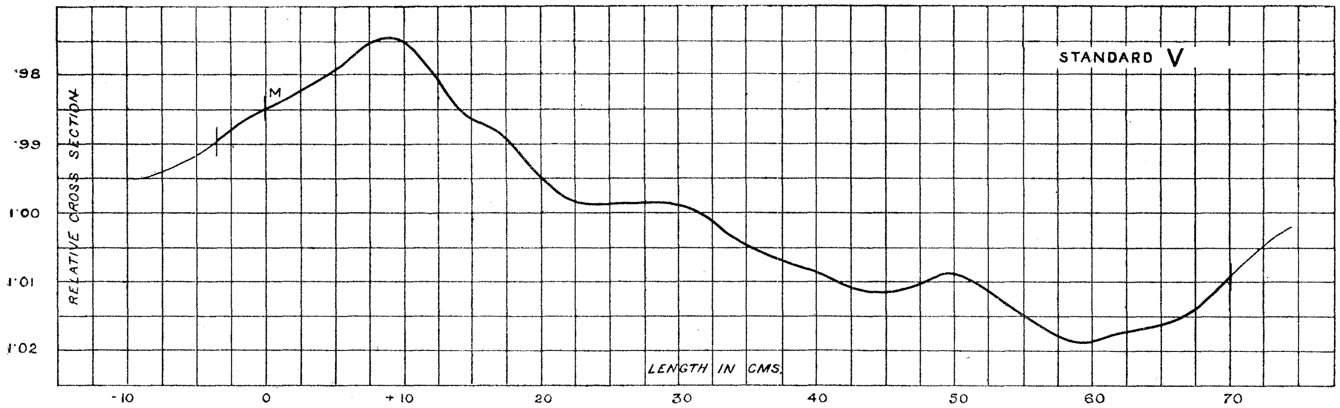
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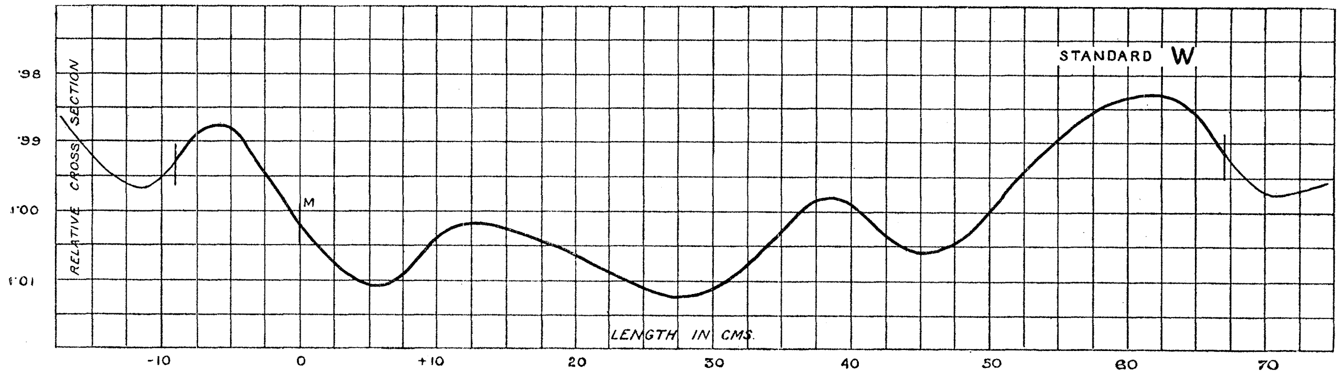
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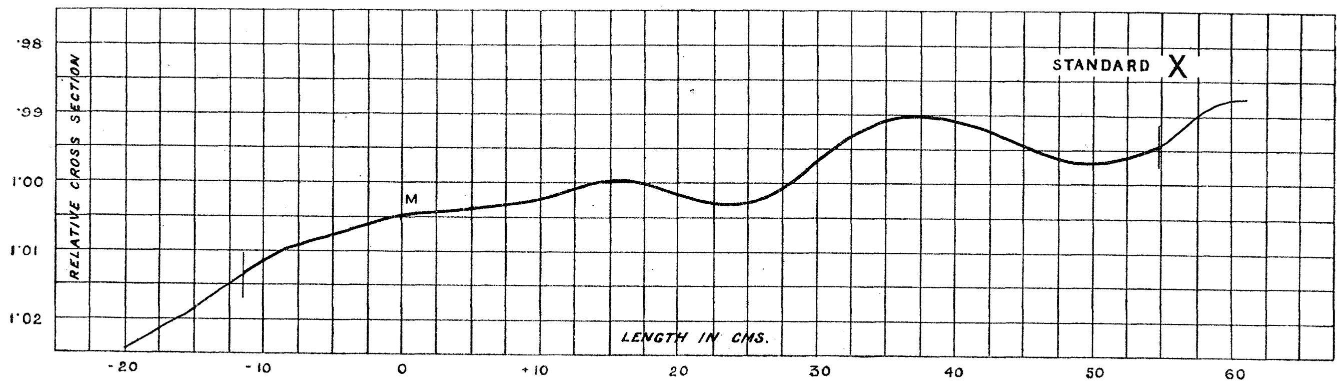
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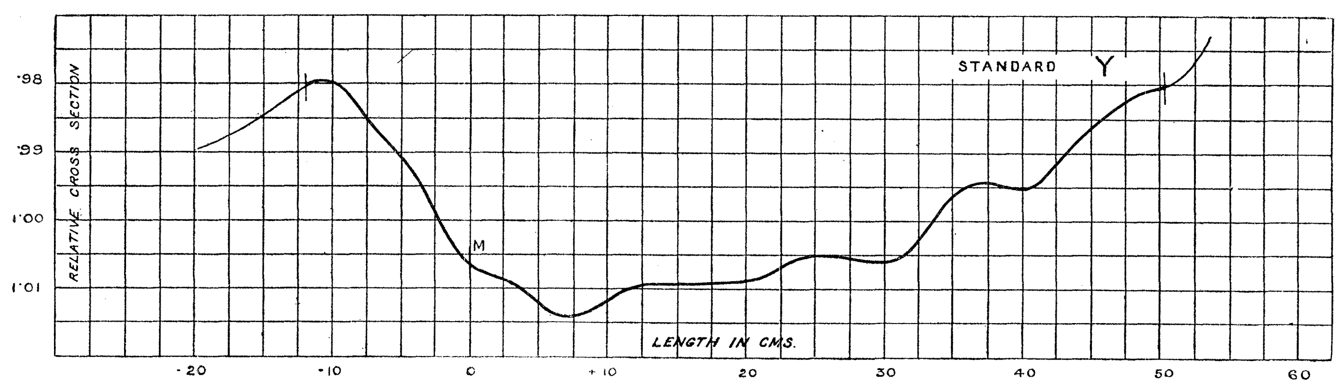
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